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ABSTRACT

Prepared for use with thirteen modules (CE 025 468-480) in the Unified Technical Concepts postsecondary physics instructional package, this set of eight support modules is designed to strengthen mathematical and laboratory skills in areas such as units, graphing, logarithms, dimensional analysis, and basic trigonometry. Module titles include Formula Interpretation: International System of Units: Logarithms and Exponents: Angles and Triangles: Vectors and Scalers: Dimensional Analysis: Reading and Drawing Graphs: and Precision, Accuracy, and Measurement. Each module follows a typical format that includes an introduction, prerequisites, objectives, discussion, summary, exercises, self-check, and references. (LRA)

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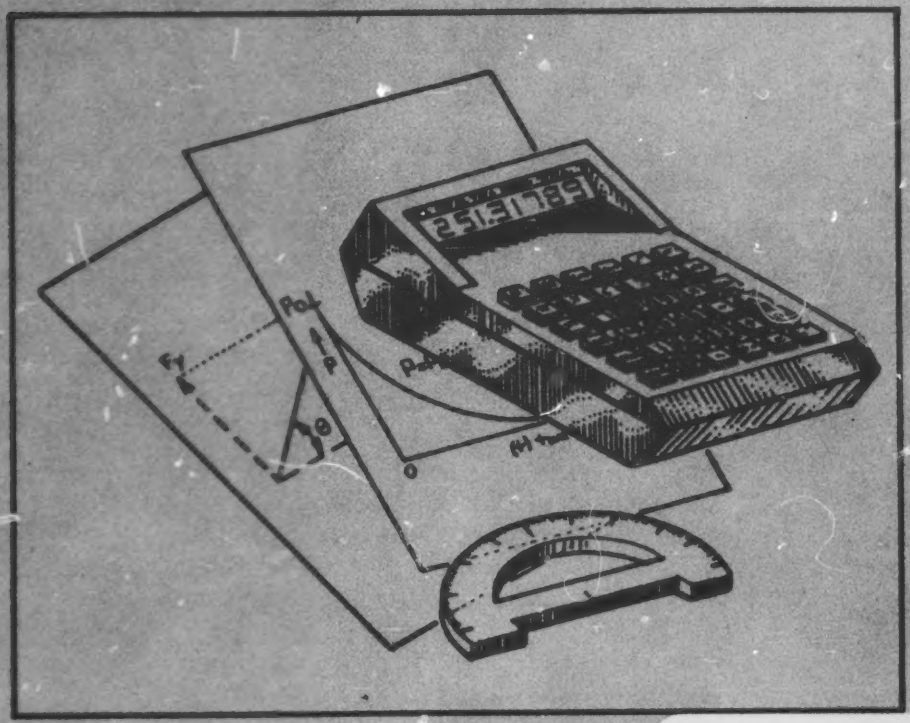
# UTC

UNIFIED TECHNICAL CONCEPTS

MODULE S 1

# FORMULA INTERPRETATION

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TECHNICAL EDUCATION RESEARCH CENTER - SW
4201 LAKE SHORE DRIVE, SUITE 111
WACO, TEXAS 76710

## INTRODUCTION TO UTC

Complex, modern equipment often contains combinations of hydraulic, pneumatic, thermodynamic or optical subsystems, as well as mechanical and electrical components. These subsystems and components appear not in isolation but as interdependent parts of complex systems. Consequently, a modern technician must understand not only the separate subsystems but also their interrelationships and the roles they play in an integrated system.

Since physics provides the foundation for modern technology, physics instruction in technical programs cannot be overemphasized. Physics courses for technicians should achieve three major objectives:

- Provide students with all the basic physics principles that apply to their chosen field
- Reinforce these principles through the use of "real" examples and hands-on experience.
- Organize knowledge and skills gained in a manner that provides ease of application in solving systems-oriented problems.

These objectives must be accomplished in a minimum of instructional time, and in a manner flexible enough to meet the needs of a variety of technical specialties.

Unified Technical Concepts (UTC) is a flexible, laboratory-based physics instructional package designed to meet the specialized needs of students in two-year, post-secondary technical schools. UTC organizes technical physics into 13 basic concepts (Force, Work, etc.) that apply in four "energy systems" (Mechanical, Fluidal, Electrical, Thermal). Each of the 13 "concept" modules discusses a single physics concept and how it is applied to each energy system. Similarly, each "application" module discusses a specific application of a single concept in a single energy system, and contains a student laboratory exercise.

The recommended prerequisite for UTC is one year of high school algebra, but eight supplementary modules are available for students requiring additional skills or review in the areas of math, units and graphing.

UTC is an alternate method of presenting what heretofore has been a formidable and sometimes confusing subject for technicians. It provides laboratory experiments that apply to specific technological fields, which allows a student to relate more quickly to module content - to understand how it applies to a specific technology.

UTC also represents a unique approach to teaching applied physics. Its flexible modular format allows it to be tailored by physics and technology instructors to meet

specific needs of students in a specific technology.

## STRUCTURE OF UTC

The modern world is based on the control and application of energy. Technology is the means by which that energy is applied to accomplish a desired task. Technical physics, being primarily a study of the physical basis of the control and use of energy, can be categorized according to the type of energy involved. Thus, in UTC, physics is divided first into four "energy systems" - Mechanical, Fluidal, Electrical and Thermal.

The same divisions are characteristic of traditional approaches to teaching technical physics. A student usually begins by studying mechanics. Once this branch of physics (energy system) is mastered, electromagnetism, thermodynamics and hydraulics are studied (one energy system at a time).

UTC takes a different approach.

### CONCEPTS IN PHYSICS

The basic ideas, or "concepts," of physics appear in all energy systems, and these concepts relate to one another in similar ways in all systems. UTC unifies technical physics by presenting first the physics concept and then its application in each energy system.

The first concept studied, for example, is that of force, - the physical quantity that produces motion. The basic characteristics of force are followed by a discussion of how this concept applies to "forcelike quantities" in the following energy "systems":

- In mechanical systems force and torque cause motion of mass.
- In fluidal systems pressure difference causes motion of fluids.
- In electrical systems potential difference causes motion of mass.
- In thermal systems temperature difference causes motion of heat energy.

Problems, demonstrations and hands-on laboratory exercises are used to show the application of the concept in each energy system, and to specific devices or processes common in modern technology.

Thus, UTC provides a student with the basic concepts of physics reinforced by practical application and organized into a consistent whole; UTC is readily applied to future technical problems, whether in other courses or on the job.

## COMPONENTS OF UTC

UTC incorporates three types of modules.

### CONCEPT MODULES

Thirteen concept modules form the backbone of UTC (each presents a single concept in applied physics and discusses the application of that concept in each energy system):

1-0 Force.
2-0 Work.
3-0 Rate.
4-0 Momentum.
5-0 Resistance.
6-0 Power.
7-0 Potential and Kinetic Energy.
8-0 Force Transformers.
9-0 Energy Convertors.
10-0 Transducers.
11-0 Vibrations and Waves.
12-0 Time Constants.
13-0 Radiation.

### APPLICATION MODULES

Each of the 170 application modules discusses the application of a single physics concept in a single energy system, and each module includes a laboratory exercise in which students investigate the application. For example, 5E5 "Resistance of Wires" (located under Concept 5-0 "Resistance"), discusses the electrical resistance of wires. In the laboratory, students calculate and measure experimentally the resistance of various wires.

### SUPPLEMENTARY MODULES

In UTC, mathematics has been reduced to a minimum: one year of high school algebra is adequate preparation. The following eight supplementary modules are available for students requiring additional preparation or review:

S1 Formula Interpretation.
S2 International System of Units.
S3 Logarithms and Exponents.
S4 Angles and Triangles.
S5 Vectors and Scalars.
S6 Dimensional Analysis.
S7 Reading and Drawing Graphs.
S8 Precision, Accuracy, and Measurement.

## STRUCTURE OF A UTC MODULE

Each module in UTC contains seven basic elements:

**INTRODUCTION**

A brief description of the topics to be discussed, and their importance in technology, introduces each module.

**PREREQUISITES**

A list of prerequisite UTC modules, and notice of any special knowledge or skills required for successful completion of the module, are given.

**OBJECTIVES**

Specific tasks that a student should be able to accomplish after completing the module are stated in behavioral terms; objectives also are the basis of testing for UTC.

**DISCUSSION**

This segment is an explanation of the topic to be studied. It contains the appropriate physics as well as examples of its application. The discussion includes development of equations, example problems, figures and tables of useful data. It addresses each of the objectives (except those pertaining directly to a laboratory experiment).

**LABORATORY (called DEMONSTRATIONS in the Concept Modules)**

The laboratory section of UTC modules consists of the following three parts:

- "Materials." A list of all equipment and supplies required for the laboratory.
- "Procedure." A step-by-step set of instructions for performing the experiment.
- "Data." An organized format for recording all necessary data and observations.

**SELF CHECK**

A set of questions and problems based on the module objectives is included. It is intended for use by students to check their own understanding of module content.

**REFERENCES**

This list of sources directs students to additional information on topics discussed in the module.

TABLE OF CONTENTS


| | |
|---|---|
| S1 | Formula Interpretation |
| S2 | International System of Units |
| S3 | Logarithms and Exponents |
| S4 | Angles and Triangles |
| S5 | Vectors and Scalars |
| S6 | Dimensional Analysis |
| S7 | Reading and Drawing Graphs |
| S8 | Precision, Accuracy, and Measurement |

## INTRODUCTION

Formulas and equations are a fundamental part of the language of science. Without them, description of the relationships among physical quantities in any precise way would be almost impossible. Technicians must know how to use them correctly.

This module discusses how to understand and to manipulate the algebraic form of an equation and to apply that equation to a physical problem. In the laboratory, the student will solve equations for indicated variables.

## PREREQUISITES

Student should have completed one year of high school algebra.

## OBJECTIVES

Upon completion of this module, the student should be able to:

- Solve a simple algebraic equation or formula for any indicated variable, using combinations of mathematical operations of addition, subtraction, multiplication, division, raising to a power, and extracting a root.

  [Example: Given the below equation, solve for A, B, C, D, and F:

  $$F^2 = \frac{AB}{C} - \sqrt{D}$$ ]

## DISCUSSION

Students of science soon learn that as their study of scientific principles progresses, more mathematics is encountered in the form of equations and formulas. Scientific facts and theories ultimately are expressed in equation form. Scientists, engineers, and technologists must be skilled in the mathematical principles used as tools of their work. To comprehend scientific principles two major tasks are required. The first is to learn what the physical quantities mean and how they are measured. The second is to learn the relationship that exists among the physical quantities — usually a quantitative relationship expressed through symbols and an equation. If the first of these tasks were accomplished without the second, one could determine what the scientific principle was about, but not what it says about the behavior of nature. Accomplishment of the second without the first would be an exercise in pure mathematics, since the equations would not be interpretable. An understanding of the equations and of the meaning of the symbols which comprise them is the essence of understanding a scientific principle.

Equations and formulas are used to express the relationships among several physical quantities, as shown in Equation 1, which relates the concepts of force, pressure, and area.

$$\text{Force} = \text{Pressure} \times \text{Area}. \qquad (1)$$

This equation provides a means of determining the value of one physical quantity if the numerical values of the other two are known. Frequently equations are written symbolically, as in Equation 2.

$$\text{In Symbols:} \quad F = pA \qquad (2)$$

(In Words, Force = Pressure x Area)

When two algebraic quantities are written together, with nothing between them, they are to be multiplied together.

Symbolic equations often are used, because they are compact, take less time to write, and are in a form for easy algebraic manipulation. Note that the symbols standing for the physical quantity often are the first letter of the physical quantity, although this is not essential. The symbol chosen to represent a physical quantity in an equation of formula is not important, but each symbol must be related clearly to a specific physical quantity.

A knowledge of the units of measure in which physical quantities are expressed is important. Some formulas in handbooks and technical material are often written so that only certain units of measure of the physical quantities involved will give the correct result when applying the formula. For example, the acceleration

of gravity (symbolically "g") in the International System of Units (SI) has a value of 9.8 $m/s^2$ and in the English system, a value of 32 $ft/s^2$. Thus, the choice of "g" value used in Equation 3 depends upon the system of units chosen for the expression of weight and mass.

$$\underline{\text{Weight}} = \underline{\text{Mass}} \times \text{acceleration of } \underline{\text{gravity}} \qquad (3)$$

In Symbols: $W = mg$

The scientist, engineer, and technician often must change formulas and equations to other desired forms by using the principles of algebra. When formulas are applied to different physical problems, they usually are changed to fit the problems. The relationship between the physical quantities is not changed. Only the algebraic form of that relationship is changed. To understand the purpose of this change of form, three concepts must be understood: the concept of a constant, an independent variable and a dependent variable. In an equation, a constant has the same value in any form of the equation. An example of a constant concept is "g" in Equation 3 or $2\pi$ in Equation 4. A "physical constant" such as g depends upon the system of units employed; whereas, a "mathematical constant" such as $2\pi$ always has the same value. The independent variable is a physical quantity whose value is known or measured in a physical problem. The dependent variable is the unknown in the physical problem, to be determined from the equation. The goal of changing the algebraic form of an equation is to isolate the dependent variable on one side of the equation and the constants and independent variables on the other side, as shown in Figure 1.

| Desired Form of Formula for Physical Problem: |
| --- |
| Dependent variable = Mathematical combinations of constants and independent variables |

FIG. 1

As an example, the period of a simple pendulum is given by Equation 4.

$$T = 2\pi\sqrt{\frac{\ell}{g}} \qquad (4).$$

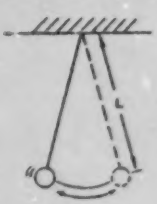
where

$\ell$ = length.

g = acceleration of gravity.

Examples A, B, and C demonstrate how Equation 4 is changed algebraically to be applied to three different problems.

**EXAMPLE A: SOLVING FOR DEPENDENT VARIABLES**




Given: A pendulum exhibits simple harmonic motion and has a length of $\ell = 0.4$ m ($g = 9.8$ m/s$^2$).

Find: The period of oscillation (T).

Solution: Constants: $g = 9.8$ m/s$^2$, $2\pi$.
Independent variable: $\ell$.
Dependent variable: T.

Desired form of Equation 4 applicable to this problem:

$$T = 2\pi \frac{\ell}{g}$$ (No change is necessary.)

**EXAMPLE B: SOLVING FOR DEPENDENT VARIABLES**

Given: A pendulum exhibits simple harmonic motion and has a length ($\ell$) of 1 m and an oscillation period (T) of 2 seconds.

Find: The acceleration due to gravity.

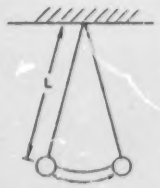
Solution: Dependent variable: g.
Constants: $2\pi$.
Independent variables: $\ell$, T.

Desired form of Equation 4 applicable to this problem:

$$g = \frac{4\pi^2 \ell}{T^2}$$

**EXAMPLE C: SOLVING FOR DEPENDENT VARIABLES**

**Given:** A pendulum exhibits simple harmonic motion and has a period (T) of 4 seconds. ($g = 9.8\ m/s^2$).




**Find:** The length ($\ell$) of the pendulum.

**Solution:** Constants: $2\pi$, $g = 9.8\ m/s^2$.
Independent variable: T.
Dependent variable: $\ell$.

Desired form of Equation 4 applicable to this problem:

$$\ell = \frac{gT^2}{4\pi^2}$$

All of the different forms in the preceding Examples can be derived from Equation 4 by using the rules of algebra. Notice that the desired dependent variable governs the equation form. Algebraically speaking, Equation 4 was solved for the desired, dependent variables or particular problems.

## SOLVING FORMULAS AND EQUATIONS

The ability to solve equations is an essential skill in the study of science, engineering, and mathematics. To "solve" an equation means to isolate a particular variable in an equation on one side of the equation by itself. To accomplish this isolation, certain mathematical operations are performed on both sides of the equation. In the performance of these "operations," certain fundamental properties, or laws of numbers, are used. These fundamental laws are known as "the associative law," the commutative law," and "the distributive law." They are illustrated in Figure 2.

| Laws | Examples |
|---|---|
| Associative: $a + (b + c) = (a + b) + c$ | $3 + (4 + 7) = (3 + 4) + 7$<br>$3 + 11 = 7 + 7$ |
| Commutative: $a + b = b + a$ | $5 + 8 = 8 + 5$ |
| Distributive: $(a + b)\,c = ac + bc$ | $(2 + 3)\,6 = 2(6) + 3(6)$ |
| where a,b,c are real numbers | $30 = 12 + 18$ |

FIG. 2

The operations performed on the formulas yield equivalent equations. Any operation performed on one side of the equation always is performed on the other side. Many students neglect this principle. Care must be taken in these areas:

- Add or subtract the same quantity to both sides of an equation.
- Multiply or divide both sides by the same quantity.
- Raise both sides to the same power.
- Extract the same root from both sides.

Frequently, a particular sequence of operations is required to solve an equation. The selection of the order in which the operations are performed is not a haphazard one. There are good reasons for a particular sequence of operations performed on both sides of the equation. Sometimes, there may be more than one order of operation from which to choose, but each algebraically-valid ordering of a problem is equivalent and will yield the same solution. The solutions produced from different orders may appear in different forms, but always can be shown to be equivalent.

Several principles must be followed in order to determine which operations to accomplish first.

- Simplify the equation by removing parentheses and other signs of grouping. Sometimes this removal is accomplished using the distributive law of algebra; sometimes, by multiplying or dividing both sides of the equation by the same factor. If fractions are involved, both sides of the equation should be multiplied by a common denominator. If radicals are involved, eliminate by raising each side of the equation to the appropriate power. Like terms should be collected if needed.
- Add the same terms to both sides of the equation or subtract them from both sides. In this way, all terms involving the "solution" variable are transposed to one side of the equation, and all other terms are moved to the other side.
- At this point, factoring may be indicated to obtain the deponent variable multiplied by a group of terms. In any event, at this point in the solution, there should be an equation similar to the general form found in Figure 3.

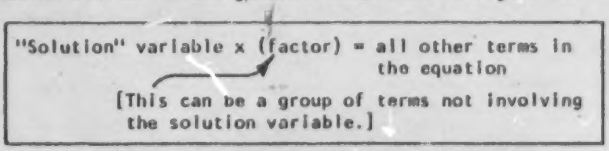

"Solution" variable x (factor) = all other terms in the equation
[This can be a group of terms not involving the solution variable.]

FIG. 3

- Both sides of the equation should be divided by the factor to obtain the solution variable alone on one side of the equation.

The proper performance of these maneuvers in isolating the dependent variable is grasped more easily through the study of specific examples. Examples D and E illustrate the preceding steps for solving equations.

**EXAMPLE D: SOLVING AN EQUATION**

Given: $I = \frac{En}{R + nr}$

Find: E.

Solution: In solving for E, write the given equation:

(1) $I = \frac{En}{R + nr}$

(2) Multiply both sides of the equation by the factor $(R + nr)$:

$$(R + nr)\, I = \cancel{(R + nr)}\left(\frac{En}{\cancel{R + nr}}\right)$$

$$(R + nr)\, I = En$$

(3) Divide both sides by n:

$$\frac{(R + nr)\, I}{n} = \frac{E\cancel{n}}{\cancel{n}}$$

$$\frac{(R + nr)\, I}{n} = E$$

**EXAMPLE E: SOLVING AN EQUATION**

Given: The equation, $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

Find: p.

Solution: (1) Multiply both sides of the equation by the least common denominator, f p q:

$$\cancel{f}\, p\, q\left(\frac{1}{\cancel{f}}\right) = f\,\cancel{p}\, q\left(\frac{1}{\cancel{p}}\right) + f\, p\,\cancel{q}\left(\frac{1}{\cancel{q}}\right)$$

(2) Cancellation gives this result:

$$p\, q = f\, q + f\, p$$

(3) Subtract f p from both sides of the equation:

$$p\, q - f\, p = f\, q + \underbrace{f\, p - f\, p}_{0}$$

$p\,q - f\,p = f\,q$

(4) Factor p in the left-hand side:

$p\,(q - f) = f\,q$

solution variable — factor — all other terms of equation

(5) Divide both sides by the factor $(q - f)$:

$$\frac{p\,(q - f)}{(q - f)} = \frac{f\,q}{(q - f)}$$

(6) These steps result in the following:

$$p = \frac{f\,q}{q - f}$$

For several reasons, students often fail to achieve an accurate solution. One common error is failure to perform the same operation to both sides of an equation. Other errors are misapplication of the distributive law by failure to distribute a factor completely through every term, thus getting the steps for solution out of order and leaving the unknown as part of the solution. There generally are several different procedures possible in isolating the dependent variable, although one method often may be more efficient.

## APPLICATION OF FORMULAS TO PHYSICAL PROBLEMS

Many formulas and equations in physics are developed within the framework of certain basic assumptions. If a particular physical problem does not fit the assumptions on which the formula is based, then the formula cannot be used to give valid results for that problem. For example, earlier in this module three pendulum problems were used to illustrate how the same equation is applied in different algebraic forms. In each problem the pendulum was assumed to undergo simple harmonic motion; therefore, Equation 3 was valid only for that assumption. If a certain pendulum under study does not exhibit simple harmonic motion, then Equation 3 cannot be used to describe correctly the motion of the pendulum in that situation. Assumptions upon which a formula or equation is based always must be considered in a physical-problem situation before a particular formula is applied to solve that problem.

## SUMMARY

Mathematical equations are statements of equality. They involve two or more numbers or functions that may be known or unknown quantities. Formulas and equations contain the symbols required for the calculation of simple or complex mathematical problems. They often must be rearranged to solve for any unknown quantity with them. The three laws of numbers employed in the solving process are (1) the associative law, (2) the commutative law, and (3) the distributive law. If a change is made to one side of an equation, it must be made to the other side. The technician must be able to use formulas and equations in the solving of problems.

## LABORATORY

### SOLVING OF EQUATIONS FOR INDICATED VARIABLES

**MATERIALS**

Pencil.
Paper.

**PROCEDURE**

Solve the following equations for the indicated variables. Write the equation and each step in the solution, showing cancellation steps. Rewrite a "clean" equation before proceeding to each step of the solution. Before each step, include a sentence that describes the operations applied to the equation. (Refer to Examples D and E for desired form.)

1. $E_p = mgh$ Solve for m.

2. $\frac{F_1}{F_2} = \frac{d_2}{d_1}$ Solve for $d_1$.

3. $\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$ Solve for $r_1$.

4. $R = R_o (1 + \alpha t)$ Solve for $\alpha$.

5. $g = \frac{4\pi^2 \ell}{T^2}$ Solve for $\ell$.

6. $T = 1/2 (P + p)S + B$ Solve for S.

[Answers to laboratory exercises are given in Self Check.]

7. What are the three laws of numbers? How many were used in this exercise?

## SELF CHECK

The following equations are laws or definitions taken from science and engineering. Solve for the unknown variables indicated.

a. $V = IR$ Solve for R.

b. $a = \frac{v_2 - v_1}{t}$ Solve for $v_2$.

c. $V^2 = V_o^2 + 2as$ Solve for s.

d. $T = \sqrt{2\,h/g}$ Solve for h.

e. $(m - ut)a = uv$ Solve for m.

f. $E_K = 1/2\ mV^2$ Solve for V.

g. $E_K = 1/2\ mV^2 + 1/2\ I\omega^2$ Solve for V.

h. $w^2 = \frac{2\mu h}{mr^2 + J}$ Solve for r.

## ANSWERS

### Answers to Laboratory Exercises

1. $m = \frac{E_p}{gh}$

2. $d_1 = (\frac{F_2}{F_1})\ d_2$

3. $r_1 = \frac{I_2}{I_1}\ r_2$

4. $\alpha = \frac{R - R_o}{R_o t}$

5. $\ell = \frac{g\ T^2}{4\pi^2}$

6. $S = \frac{2(T - \theta)}{(P - \rho)}$

### Answers to Self Check Exercises

a. $R = \frac{V}{I}$

b. $v_2 = at + v_1$

c. $s = \frac{v^2 - v_o^2}{2a}$

d. $h = \frac{T^2 g}{2}$

e. $m = \frac{uv}{a} + ut$

f. $V = \frac{2E_K}{m}$

g. $V = \sqrt{\frac{2E_K - I\omega^2}{m}}$

h. $r = \sqrt{\frac{2\mu h - w^2 J}{mw^2}}$

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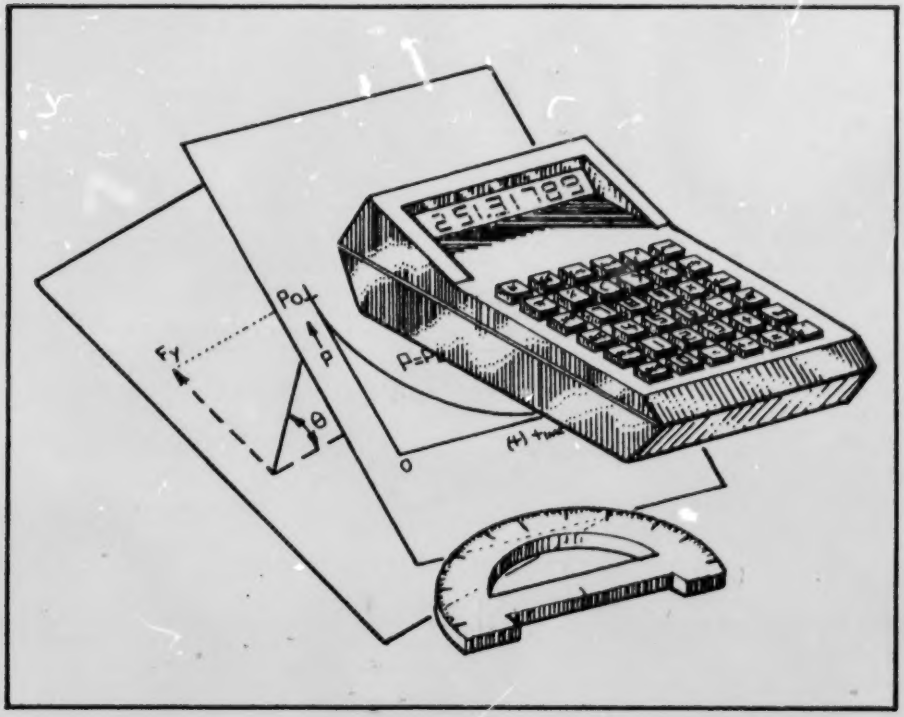
UTC

UNIFIED TECHNICAL CONCEPTS

MODULE S 2

INTERNATIONAL SYSTEM OF UNITS




SUPPLEMENTARY

Technical Education Research Center - Southwest TERC - SW

## INTRODUCTION

Physics is an attempt to explain, in precise terms, the interactions of matter and energy. This attempt requires a consistent system of units for measuring and describing all the properties of matter and energy. The International System of Units (abbreviated SI for the French name Le Systéme International d'Unités) is the accepted system of units for science and technology throughout the world. This module discusses the units within the SI system, prefixes used with these units, proper procedure for writing SI units, and conversions from SI units to English units.

## PREREQUISITES

Student should have completed Support Module S1, "Formula Interpretation." One year of high school algebra is recommended.

## OBJECTIVES

Upon completion of this module, the student should be able to:

- Name the seven base SI units, and give the symbol and quantity measured for each.
- Name the two supplementary SI units, and give the symbol and quantity measured for each.
- List the derived SI unit and its symbol for each of the following quantities:

  Frequency.
  Force.
  Energy.
  Power.
  Electrical charge.
  Electrical potential.
  Electrical resistance.

- State the name and symbol of the prefix for each of the following multiples: $10^{12}$, $10^{9}$, $10^{6}$, $10^{3}$, $10^{-2}$, $10^{-3}$, $10^{-6}$, $10^{-9}$, $10^{-12}$, $10^{-15}$, $10^{-18}$.

- Given quantities expressed in SI units, use prefixes to write an expression for the same quantity with the numerical values between 0.1 and 1000.

- Write, in the accepted notation, expressions indicating the products, quotients, and reciprocals of given SI units.

- Name one non-SI unit that may be used with SI units for measuring the following quantities:

  Time.
  Plane angle.
  Volume.
  Mass.

- Convert lengths in feet and inches to the equivalent length in SI units.

- Given a mass in kilograms, calculate the weight in pounds.

## ORIGIN OF THE INTERNATIONAL SYSTEM OF UNITS

The International System of Units, a concise set of units allowing measurement of all physical quantities, was adopted by the International Organization of Standardization in 1972. The basis of the SI system is the older metric system. This system was chosen because it is a decimal system; that is, each unit for a particular quantity is related to all other units for that quantity by a multiple of ten.

All standards of measurement are arbitrary. Their virtue lies in the fact that they are useful, reproducible, and accepted by those who use them. The original meter was intended to be one ten-millionth the distance from the north pole to the equator, along a line passing through Paris. This distance was indicated by two fine lines scribed on gold plugs in an X-shaped platinum-iridium bar. When this bar is at 0°C (the temperature of melting ice), the distance between the marks is defined as one meter. This distance actually is different from the original intended distance by 0.023%; but this is unimportant, because the standard is arbitrary.

The standard kilogram originally was intended to be the mass of 1000 $cm^3$ of pure water at 4°C. Once again, this measurement is not exactly accurate. One kilogram is defined to be equal to the mass of a platinum-iridium cylinder called the "standard kilogram," and carefully preserved. The second was chosen arbitrarily as 1/86,400th the duration of the average solar day. Modern definitions of the meter and of the second are more precise; they are given later in this module.

Several other quantities besides length, mass, and time must be measured for a complete description of the physical world. There are many ways of defining basic units. All electrical units, for example, could be defined in terms of voltage, current, or charge. In the SI system, current has been chosen arbitrarily as the fundamental quantity. Once a system of base units has been established, all other necessary units can be derived from them. In any measurement system, the base units are arbitrary.

## UNITS IN THE SI SYSTEM

There are three classifications of units in the SI system. The base units are given in Table 1, along with their symbols and the fundamental quantities they are used to measure. The definition of each base unit is completely arbitrary and is independent of all other base units. Their exact definitions appear later in this module.

| TABLE 1. SI BASE UNITS | | |
|---|---|---|
| Quantity | Name of base SI unit | Symbol |
| length | meter | m |
| mass | kilogram | kg |
| time | second | s |
| electric current | ampere | A |
| thermodynamic temperature | kelvin | K |
| amount of substance | mole | mol |
| luminous intensity | candela | cd |

Table 2 lists the two supplementary units of the SI system. These units are used to measure plane and solid angles and do not meet the exact criteria for either base or derived units.

| TABLE 2. SI SUPPLEMENTARY UNITS | | |
|---|---|---|
| Quantity | Name of Supplementary SI Unit | Symbol |
| plane angle | radian | rad |
| solid angle | steradian | sr |

A large number of units can be derived from the base and supplementary SI units. Table 3 lists the derived units of the SI system that have special names. Many other common derived units do not have specific names; examples are velocity, measured in meters per second (m/s), and volume, measured in cubic centimeters ($cm^3$).

| TABLE 3. SI DERIVED UNITS FROM NAMES | | | |
|---|---|---|---|
| Quantity | Name of derived SI unit | Symbol | Expressed in terms of base or supplementary SI units or in terms of other derived SI units |
| frequency | hertz | Hz | $1\ Hz = 1\ s^{-1}$ |
| force | newton | N | $1\ N = 1\ kg \cdot m/s^2$ |
| pressure, stress | pascal | Pa | $1\ Pa = 1\ N/m^2$ |
| energy, work, quantity of heat | joule | J | $1\ J = N \cdot m$ |
| power | watt | W | $1\ W = 1\ J/s$ |
| electric charge, quantity of electricity | coulomb | C | $1\ C = 1\ A \cdot s$ |
| electrical potential, potential difference, tension, electromotive force | volt | V | $1\ V = 1\ J/C$ |
| electric capacitance | farad | F | $1\ F = 1\ C/V$ |
| electric resistance | ohm | Ω | $1\ \Omega = 1\ V/A$ |
| electric conductance | siemens | S | $1\ S = 1\ \Omega^{-1}$ |
| flux of magnetic induction, magnetic flux | weber | Wb | $1\ Wb = 1\ V \cdot s$ |
| magnetic flux density, magnetic induction | tesla | T | $1\ T = 1\ Wb/m^2$ |
| inductance | henry | H | $1\ H = 1\ Wb/A$ |
| luminous flux | lumen | lm | $1\ lm = 1\ cd \cdot sr$ |
| illuminance | lux | lx | $1\ lx = 1\ lm/m^2$ |

The prefixes given in Table 4 are an important part of the SI system. They allow any numerical value to be written as a number between 0.1 and 1000, as shown, for example, in Table 5. This may at first seem a trivial convenience, but it actually is one of the most important characteristics of the SI system. The most important prefixes in Table 4 are indicated by an asterisk. Centi ($10^{-2}$) is used primarily with meter to form the useful centimeter. Table 5 shows the use of these prefixes. Notice that in some cases there are two acceptable statements of the same quantity (0.325 mm and 325 μm). The kilogram is a special case. Because the name of this base unit of mass contains the name of the SI prefix "kilo," the names of the decimal multiples and submultiples of the unit of mass are formed by adding the prefixes to the word "gram." Thus, $10^{-6}$ kg is written as milligram (mg) instead of micro-kilogram (μkg).

TABLE 4. SI PREFIXES

| Factor by which the unit is multiplied | Prefix | |
|---|---|---|
| | Name | Symbol |
| $10^{12}$ | tera | T |
| * $10^{9}$ | giga | G |
| * $10^{6}$ | mega | M |
| * $10^{3}$ | kilo | k |
| $10^{2}$ | hecto | h |
| 10 | deca | da |
| $10^{-1}$ | deci | d |
| * $10^{-2}$ | centi | c |
| * $10^{-3}$ | milli | m |
| * $10^{-6}$ | micro | μ |
| * $10^{-9}$ | nano | n |
| $10^{-12}$ | pico | p |
| $10^{-15}$ | femto | f |
| $10^{-18}$ | atto | a |

| TABLE 8. USE OF SI PREFIXES | | | | |
|---|---|---|---|---|
| Quantity Expressed in Base or Derived Units | | Quantity Restated in Base or Derived Units | | Quantity Expressed with prefix and base of derived unit |
| 12000 N | | $12 \times 10^{3}$ N | | 12 kN |
| 0.000325 m | or | $0.325 \times 10^{-3}$ m<br>$325 \times 10^{-6}$ m | or | 0.325 mm<br>325 μm |
| 1401 V | | $1.401 \times 10^{3}$ V | | 1.401 kV |
| 34200 W | | $34.2 \times 10^{3}$ W | | 34.2 kW |
| $1.87 \times 10^{-7}$ S | or | $0.187 \times 10^{-6}$ S<br>$187 \times 10^{-9}$ S | or | 0.187 μs<br>187 ns |
| $3.28 \times 10^{7}$ J | | $32.8 \times 10^{6}$ J | | 32.8 MJ |
| 0.95 kg | | $950 \times 10^{-3}$ kg | | 950 g |

## RULES FOR WRITING SI UNITS

The following are rules for writing SI units:

- The unit symbols are written in lower case letters, except that the first letter is written in upper case when the unit is derived from a proper name.

  Examples: m meter A ampere
  s second Wb weber

- Compound units formed by the multiplication of two or more units may be written in one of the following ways:

  N·m N.m Nm.

  The first of these is most common and will be used throughout UTC to eliminate possible confusion. (Notice that mN could be read as millinewton or meter times newton if the dot indicating multiplication is not used consistently. Such ambiguity must be avoided.)

- Compound units formed by the division of one unit by another are written in the following ways:

  m/s $\frac{m}{s}$ $m \cdot s^{-1}$

  The notation $s^{-1}$ is the same as 1/s and indicates the reciprocal of the base or derived unit.

- In no case should more than one solidus (/ as in m/s) on the same line be included in such a combination unless parentheses are included to avoid all ambiguity. Powers of units may also be used:

  Example: unacceptable — m/s/s

  acceptable — (m/s)/s $m/s^2$ $m \cdot s^{-2}$

- Only one prefix may be used when forming a multiple or submultiple of a base or derived unit.

  Example: unacceptable — mμm (millimicrometer)

  acceptable — nm (nanometer)

Example A shows several improper expressions written in SI units and a better statement of each quantity. Such "improper" expressions are in common use and some will probably remain so, because they are convenient in some calculations.

**EXAMPLE A: WRITING QUANTITIES WITH SI UNITS**

Given: The following table containing quantities expressed improperly in SI units.

Find: Proper expression for each quantity.

Solution:

| Quantity Measured | Improper Expression | Proper Expression |
|---|---|---|
| length | 2480 m | 2.48 km |
| mass | 0.00032 kg | 0.32 g or 320 mg |
| time | $6.72 \times 10^{-5}$ s | 67.2 μs |
| velocity | 0.025 m/s | 2.5 cm/s or 25 mm/s |
| energy | 550 mJ | 5.5 J |
| acceleration | $4.3 \times 10^{3}$ m/s/s | $4.3$ km/s$^2$ |
| power | 2.5 kMW | 2.5 GW |

## OTHER UNITS USED WITH SI UNITS

Certain units outside the SI system frequently are used with SI units because of their practical importance (Table 6) or because of the needs of specialized fields (Table 7). In a limited number of cases, compound units are formed from these units and SI units; for example, kg/h and km/h. Such units are not strictly part of the SI system.

TABLE 6. NON-SI UNITS USED WITH SI UNITS BECAUSE OF PRACTICAL IMPORTANCE

| Quantity | Name of Unit | Unit Symbol | Definition |
|---|---|---|---|
| time | minute | min | 1 min = 60 s |
| | hour | h | 1 h = 60 min |
| | day | d | 1 d = 24 h |
| plane angle | degree | ° | $1° = (\pi/180)$ rad |
| | minute | ' | $1' = (1/60)°$ |
| | second | '' | $1'' = (1/60)'$ |
| volume | litre | l | $1 l = 1 dm^3$ |
| mass | tonne | t | $1 t = 10^3$ kg |

TABLE 7. SPECIAL UNITS USED WITH SI UNITS

| Quantity | Name of Unit | Unit Symbol | Definition |
|---|---|---|---|
| energy | electron-volt | eV | 1 electron volt is the kinetic energy acquired by an electron in passing through potential difference of 1 volt in vacuum; $1 eV = 1.60219 \times 10^{-19}$ J |
| mass of an atom | atomic mass unit | u | 1 (unified) atomic mass unit is equal to the fraction 1/12 the mass of an atom of the nuclide $C^{12}$; $1 u = 1,66053 \times 10^{-27}$ kg |

| | | | |
|---|---|---|---|
| length | astronomic unit | AU[1] | 1 AU = 149,600 x $10^6$ m (System of astronomic constants, 1964) |
| | parsec | pc | 1 parsec is the distance at which 1 astronomic unit subtends an angle of 1 second of arc; 1 pc = 206.265 AU = 30.857 x $10^{12}$ m |
| pressure of fluid | bar[2] | bar | 1 bar = $10^5$ Pa |

[1]This unit has no international symbol; AU is the abbreviation of the English name; the abbreviation of the French name is UA.

[2]The bar is not mentioned by CIPM in this group of units; in many countries, however, there are special requirements for this unit.

## SI UNITS IN UNIFIED TECHNICAL CONCEPTS

The International System of Units is used throught Unified Technical Concepts and is itself an aid to unification. Although some examples require a detailed study of the physics involved, as in Concept Module 1-0, "Force," others illustrate unification with a minimum of explanation. The following are examples:

- In Concept Module 2-0, "Work," the unit of energy in the SI system is always the Joule (J). This unit is used to express energy in mechanical, fluid, thermal, electrical, and optical systems throughout UTC.
- In Concept Module 3-0, "Rate," all rates are expressed in SI units as some quantity divided by time in seconds. Any quantity having a denominator in seconds is a rate.
- In Concept Module 6-0, "Power," power is defined as energy divided by time, or the rate of doing work, and is measured in watts (1W = 1 J/s). This definition is true in all energy systems (mechanical, electrical, etc.)

Because of the practical nature of UTC, and because SI is not the only system of units in use today, other common units are used in UTC modules. Quantities expressed in SI units often must be converted to equivalent expressions in English units. Table 8 lists some of the more useful conversion factors. The use of these conversion factors is illustrated in Example B.

| TABLE 8. CONVERSION TO SI UNITS | | |
|---|---|---|
| Quantity | Non SI Unit | SI Equivalent |
| length | 1 inch (in) | 2.54 cm |
| length | 1 yard (yd) | 0.9144 m |
| force | 1 pound (lb) | 4.445 N |
| power | 1 horsepower (hp) | 746 W |
| energy | 1 calorie (cal) | 4.186 J |

**EXAMPLE B: CONVERSION TO SI UNITS**

Given: The following quantities:

(a) 12 in
(b) 0.25 hp
(c) 250 cal

Find: Equivalent expressions in SI units.

Solution:

(a) $(12\ \text{in})\left(\frac{2.54\ \text{cm}}{1\ \text{in}}\right) = 30.48\ \text{cm}$

(b) $(0.25\ \text{hp})\left(\frac{746\ \text{W}}{1\ \text{hp}}\right) = 186.5\ \text{W}$

(c) $(250\ \text{cal})\left(\frac{4.186\ \text{J}}{\text{cal}}\right) = 1046.5\ \text{J} = 1.0465\ \text{kJ}$

In the SI system, the base unit of mass is the kilogram and the unit of force (N) is derived. The English system uses a base unit of force called the "pound" and a less-familiar derived mass unit called a "slug." There can be no conversion of pounds to kilograms, because the two units do not measure the same quantity. (The two systems were developed from two different arbitrary starting points.) One should remember, however, that one kilogram weighs 2.2 pounds on the earth's surface, as illustrated in Example C.

| EXAMPLE C: WEIGHT OF MASS IN KILOGRAMS | |
|---|---|
| Given: | Mass of 84 kg. |
| Find: | Weight in pounds. |
| Solution: | 1 kg weighs 2.2 lb |
| | $(84\ kg)(\frac{2.2\ lb}{kg}) = 184.8\ lb$ |

## DEFINITIONS OF SI UNITS

The exact definitions of most SI units are actually of little interest in UTC. They are given here to illustrate the precision with which they are defined and their arbitrary nature.

### BASE UNITS

- Meter — The meter is the length equal to 1,650,763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels $2p_{1_0}$ and $5d_5$ of the krypton-86 atom.
- Kilogram — The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.
- Second — The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.
- Ampere — The ampere is that constant electric current which, if maintained in two straight, parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to $2 \times 10^{-7}$ newton per meter of length.
- Kelvin — The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.
- Mole — The mole is the amount of substance of a system that contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.

- Candela — The candela is the luminous intensity, in the perpendicular direction, of a surface of 1/600,000 square meter of a black body, at the temperature of freezing platinum, under a pressure of 101,325 newtons per square meter.

### SUPPLEMENTARY UNITS

- Radian — The radian is the plane angle between two radii of a circle which cut off on the circumference an arc equal in length to the radius.

- Steradian — The steradian is the solid angle which, having its vertex at the center of a sphere, cuts off an area of the surface of the sphere equal to that of a square with sides of length equal to the radius of the sphere.

## SUMMARY

The International System of Units (SI) is now the accepted system of measurement units throughout the world. Based largely upon the Metric System, the SI System is most important in technology. There are base units, units derived from names, and prefixes. Other units not a part of the SI System are used with it; these units include the minute, the hour, and the day.

The seven basic SI units are the meter (symbol m), kilogram (kg), second (s), ampere (A), kelvin (K), mole (mol), and candela (cd). The radian (rad) and steradian (sr) are supplementary units. Derived units include the hertz (Hz), newton (N), joule (J), watt (W), coulomb (C), volt (V), farad (F), ohm (Ω), weber (Wb), tesla (T), henry (H), lumen (lm), lux (lx), pascal (Pa), and siemens (S).

## CALCULATIONS AND CONVERSIONS WITH SI UNITS

### MATERIALS

Calculator (recommended but not required).

### PROCEDURE

1. Use the necessary prefixes, and write the following quantities in SI units with a numerical value between 0.1 and 1000:

| | | | |
|---|---|---|---|
| $2.8 \times 10^5$ m | = _____ | 62,700,000 Ω | = _____ |
| $2.8 \times 10^5$ nm | = _____ | 0.0015 A | = _____ |
| $2.8 \times 10^5$ mm | = _____ | 0.015 cm | = _____ |
| 350,000 Hz | = _____ | $8.4 \times 10^8$ W | = _____ |
| $3.5 \times 10^9$ Hz | = _____ | $2.3 \times 10^{-6}$ kg | = _____ |
| $1.45 \times 10^{-8}$ s | = _____ | 0.04 N | = _____ |

2. Perform the following calculations, and express the result in proper SI notation:

| | |
|---|---|
| 82 m ÷ 16 s | = _____ |
| 55 mm ÷ 180 ms | = _____ |
| 1.36 kg ÷ 140 s | = _____ |
| 18 m/s x 480 s | = _____ |
| 5.50 kg x 16 mm/s | = _____ |
| 3.2 N x 6.5 m | = _____ |

3. Find the reciprocal of the following values, and express the results in proper SI notation:

| | |
|---|---|
| t = 3 s | $\frac{1}{t}$ = _____ |
| l = 0.15 cm | $\frac{1}{l}$ = _____ |
| f = 60 Hz = 60 $s^{-1}$ | $\frac{1}{f}$ = _____ |

4. Convert the following values to equivalent values expressed in SI notation:

0.002 in = _____

42 ft = _____

150 hp = _____

150 lb = _____

840 cal = _____

45 min = _____

0.25 l = _____

5. Calculate the weight of the following masses in pounds:

650 g wt. = _____ lb

400 kg wt. = _____ lb

160 mg wt. = _____ lb

## SELF CHECK

1. Complete the following chart of SI base units:

| Symbol | Name | Quantity Measured |
|---|---|---|
| s | | |
| kg | | |
| mol | | |
| A | | |
| cd | | |
| K | | |
| m | | |

2. List the quantity measured and the symbol for the following SI supplementary units:

Radian.
Steradian.

3. Complete the following chart of SI derived units:

| Symbol | Name | Quantity Measured |
|---|---|---|
| W | | |
| J | | |
| Hz | | |
| N | | |
| V | | |
| Ω | | |
| C | | |

4. State the symbol and the multiple of ten indicated by each of these prefixes:

Milli.
Micro.
Centi.
Mega.
Giga.
Kilo.
Nano.

5. Write the following quantities in proper SI notation:

4876000 Ω.

0.0027 cm.

1440 mW.

$5.2 \times 10^{4}$ g.

0.055 ms.

6. Perform the following calculations, and write the answers in proper SI notation:

15 m/s ÷ 8 s = _____.

650 kg x 3.2 m/s = _____.

0.28 kg ÷ 30 x = _____.

7. Name a non-SI unit that may be used with SI units for measuring the following quantities:

Time.
Volume.
Plane Angle.
Mass.

8. Convert these lengths to SI units:

   16 ft.
   27.5 in.
   0.025 in.

9. Calculate the weight in pounds of the following masses:

   15 kg.
   350 g.

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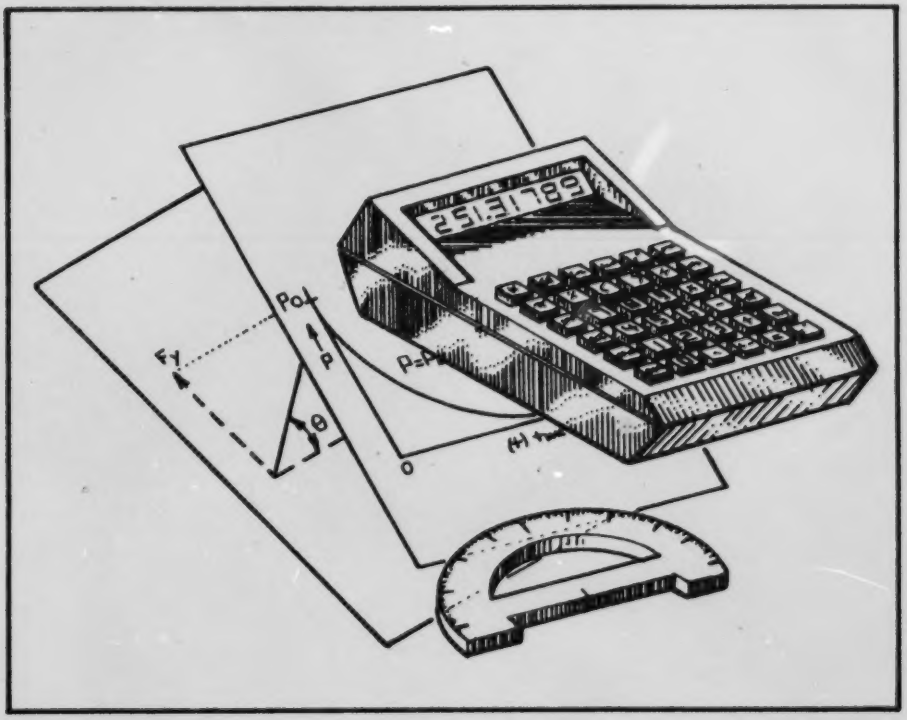
UTC

UNIFIED TECHNICAL CONCEPTS

MODULE S 3

LOGARITHMS AND EXPONENTS




SUPPLEMENTARY

Technical Education Research Center-Southwest TERC-SW

## INTRODUCTION

Scientists, engineers, and technicians work with data which range from astronomically large numbers to extremely small numbers. To express these large and small numbers efficiently, a particular notation has been adopted. This notation — known as "exponential," or "power-of-ten," notation — facilitates the recording and manipulating of numerical data.

Logarithms are important in engineering, chemistry, physics, electronics, and other fields, because many problems involving growth and decay common to these fields are exponential (logarithmic in nature). In this age of computers and electronics, difficult calculations often are done by computer. The availability of electronic calculators and computers, however, has not decreased the importance of understanding the basic tools in carrying out calculations that involve mathematical data. Embarrassment and incorrect answers can result when engineers or technicians unfamiliar with logarithmic manipulations are forced to use logarithms (perhaps because of a power failure of the calculator!) Furthermore, an engineer or technician may struggle through a laborious method of solving a problem which could have been solved easily with logarithms.

In this module, the principles of power-of-ten-exponents and logarithms will be presented so that the student can apply these principles to specific calculations. In the laboratory, the student will determine logarithms and use exponents.

## PREREQUISITES

Student should have completed one year of high school algebra.

## OBJECTIVES

Upon completion of this module, the student should be able to:

- Given a number in decimal form, express it in scientific notation. Given a number in scientific notation, express it in decimal form.

- Add, subtract, multiply, or divide given power-of-ten-numbers.

- Find logarithms and antilogarithms of given numbers, using tables and linear interpolation for common logarithms and natural logarithms.

- Using the three theorems of logarithms for products and quotients, multiply and divide numbers and solve equations.

- Given an equation containing logarithmic or exponential terms and the numerical values of all but one variable in the equation, compute (calculate) the value of the unknown variable with the aid of log tables or an electronic calculator.

## DISCUSSION

According to the Guiness Book of World Records, the remotest astronomical body that can be seen with the naked eye is in the galaxy Andromeda nebula — a distance of some 13,000,000,000,000,000,000 miles. Frequently, large and small numbers, which would require a great amount of space and effort to write in decimal notation, are encountered in technical work. The same distance mentioned previously can be written more compactly in an exponential form of notation called "power-of-ten." The distance would be written $1.3 \times 10^{19}$ miles. Representing the distance in this manner not only makes writing large and small numbers easier, but also simplifies calculations and reduces the amount of space required to report such numbers.

In order to write a power-of-ten number originally in decimal form, the number must be factored into two separate parts, just as 80 could be separated into a product of factors, 8.0 and 10 ($8 \times 10$). The first leading factor contains the significant digits of the number. Significant digits are the specified digits of a number; they indicate the accuracy of the number or measurement. The second factor following the multiplication sign (x) is a number that contains the power of ten. The power of ten contained in the second number is expressed with a small number written slightly above and to the right of the number 10. This number is called the "exponent," an example of which is illustrated in Figure 1. Two cautions should be observed when converting a number from decimal notation to power-of-ten, or vice versa. When converting from one notation to the other, no rounding off, or dropping of significant digits, should occur. The accuracy of a measurement should not change because of a change in notation in reporting the numbers. Although not a fixed rule, the first factor customarily is represented as a number between 1 and 10. In this practice, the number is expressed in a special kind of exponential notation using power-of-ten called "scientific notation."

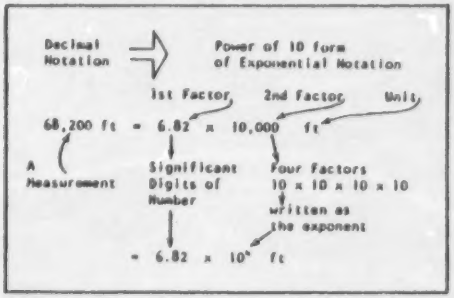

Decimal Notation
Power of 10 form of Exponential Notation
1st Factor
2nd Factor
Unit
68,200 ft = 6.82 x 10,000 ft
A Measurement
Significant Digits of Number
Four Factors 10 x 10 x 10 x 10
written as the exponent
= 6.82 x 10⁴ ft

FIG. 1

Table 1 lists several examples of power-of-ten and their decimal equivalents. A review of this table is worthwhile before proceeding to examples of conversion of numbers from decimal notation to power-of-ten. Numbers greater than 1 will have exponents of positive algebraic sign, and numbers less than 1 will have exponents with negative algebraic signs.

| TABLE 1. POWERS-OF-TEN AND DECIMAL EQUIVALENTS | | |
|---|---|---|
| Power of 10 | Decimal Equivalent | Explanation |
| $10^5$ | 100,000 | 10 x 10 x 10 x 10 x 10 |
| $10^3$ | 1,000 | 10 x 10 x 10 |
| $10^1$ | 10 | ($10^1$ is usually expressed as 10 without showing the exponent) |
| $10^0$ | 1 | BY DEFINITION - ANY NUMBER TO THE ZERO POWER EQUALS 1. |
| $10^{-1}$ | 0.1 | $\frac{1}{10}$ |
| $10^{-2}$ | 0.01 | $\frac{1}{10 \times 10}$ |
| $10^{-3}$ | 0.001 | $\frac{1}{10 \times 10 \times 10}$ |

Every decimal number can be expressed in power-of-ten notation. Consider the following examples:

- $375{,}000 = 3.75 \times 10^5$
- $81{,}000 = 8.1 \times 10^4$
- $623{,}000{,}000{,}000 = 6.23 \times 10^{11}$
- $0.0715 = 7.15 \times 10^{-2}$
- $0.000000133 = 1.33 \times 10^{-7}$

All the above examples have been written in scientific notation. Other powers-of-ten also can be used to represent the magnitude of numbers; however, the first factor must be adjusted by a factor of ten. This point is made clear by the second example above, which can be written in several powers-of-ten:

power-of-ten exponential form

$81{,}000 = 8.1 \times 10^4$ (scientific notation)
$= 81 \times 10^3$
$= 810 \times 10^2$
$= 8100 \times 10^1$
$= 0.81 \times 10^5$

In calculation with power-of-ten numbers, many instances occur in which one or more numbers might be represented best in a power-of-ten form other than scientific notation.

## ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF NUMBERS EXPRESSED IN POWERS OF TEN

The following rules must be observed when mathematical operations are conducted with numbers expressed in power-of-ten form:

Rule 1: Any number to the zero power (exponent) is equal to 1.

Examples: $10^0 = 1$, $2^0 = 1$, $5^0 = 1$.

Rules 2: Any number to the first power is equal to that number.

Exmaples: $10^1 = 10$, $2^1 = 2$, $5^1 = 5$.

Rule 3: If n is a positive number and is a non-zero number, then

$$a^{-n} = \frac{1}{a^n}.$$

Example:

$$10^{-2} = \frac{1}{10^2}$$

Rule 4: Before numbers are added or subtracted, the exponents of ten must be the same. The first factor need not be between 1 and 10.

Example 1:
$$\begin{array}{rl} 2.610 \times 10^3 = & 2.610 \times 10^3 \\ + \underline{6.2000 \times 10^1 = } & \underline{62.000 \times 10^1} \\ & 64.61 \times 10^3 \end{array}$$

or $6.461 \times 10^4$

Example 2:
$$\begin{array}{rl} 3.930 \times 10^{-6} = & 3.930 \times 10^{-6} \\ -\underline{7.81 \times 10^{-7} = } & \underline{0.781 \times 10^{-6}} \\ & 3.149 \times 10^{-6} \end{array}$$

Rule 5: In multiplication, the first factors of each number are multiplied; but the exponents of 10 are added algebraically, the result being the exponent of 10 of the product.

Example 1:
$$\begin{aligned} & (3.2 \times 10^4)(1.5 \times 10^2) \\ &= (3.2)(1.5) \times 10^{4+2} \\ &= 4.8 \times 10^6 \end{aligned}$$

Example 2:
$$\begin{aligned} & (3.2 \times 10^{-3})(1.5 \times 10^2) \\ &= (3.2)(1.5) \times 10^{-3+2} \\ &= 4.8 \times 10^{-1} \end{aligned}$$

Rule 6: In division, the first factors of each number are divided; but the exponents of 10 are subtracted algebraically, the result being the exponent of 10 of the quotient.

Example 1: $\frac{8.4 \times 10^{9}}{2.1 \times 10^{4}} = \frac{8.4}{2.1} \times 10^{9-4}$

$= 4.0 \times 10^{5}$

Example 2: $\frac{8.4 \times 10^{7}}{2.1 \times 10^{-3}} = \frac{8.4}{2.1} \times 10^{7-(-3)}$

$= 4.0 \times 10^{10}$

## LOGARITHMS

A common logarithm (log) is the exponential expression of a number having 10 as the base; that is, it is the power-of-ten required to produce that number. $\text{Log}_b$ (y) means "the power of b to produce y." When the base is not given, it is implied to be 10. An example of an equation written in exponential form is $y = b^x$. Written in logarithmic form, this equation gives $x = \log y$. Figure 2 contrasts the two different forms of equations and illustrates the basic definitions in each form.

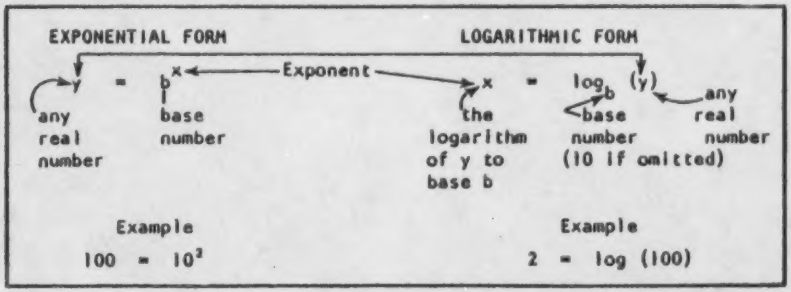



FIG. 2

Figure 3 illustrates several exponential and logarithmic forms of equations having 10 as the base. The logarithms of numbers between 1.00 and 9.99 are given in Table 2, a Table of Common Logarithms of Numbers.

| EXPONENTIAL FORM | | LOGARITHMIC FORM | | |
|---|---|---|---|---|
| 1 | $= 10^{0.00000}$ | log | 1 | = 0.00000 |
| 2 | $= 10^{0.3010}$ | log | 2 | = 0.3010 |
| 3 | $= 10^{0.4771}$ | log | 3 | = 0.4771 |
| 4 | $= 10^{0.6021}$ | log | 4 | = 0.6021 |
| 5 | $= 10^{0.6990}$ | log | 5 | = 0.6990 |
| 6 | $= 10^{0.7882}$ | log | 6 | = 0.7882 |
| 7 | $= 10^{0.8451}$ | log | 7 | = 0.8451 |
| 8 | $= 10^{0.9031}$ | log | 8 | = 0.9031 |
| 9 | $= 10^{0.9542}$ | log | 9 | = 0.9542 |
| 10 | $= 10^{1.0000}$ | log | 10 | = 1.0000 |
| 100 | $= 10^{2.0000}$ | log | 100 | = 2.0000 |
| 1000 | $= 10^{3.0000}$ | log | 1000 | = 3.0000 |
| 0.1 | $= 10^{-1}$ | log | 0.1 | = -1 |
| 0.01 | $= 10^{-2}$ | log | 0.01 | = -2 |
| 0.001 | $= 10^{-3}$ | log | 0.001 | = -3 |

FIG. 3

To find the logarithm of any number from Table 2, locate the first two digits of the number in the left hand column of the table. Next, move horizontally to the right until the column headed by the third digit of the number is encountered. The desired logarithm of the number lies at this intersection in the table. It is expressed as a decimal fraction less than 1. Example A illustrates the method of finding a logarithm in Table 2.

**EXAMPLE A: FINDING THE LOGARITHM OF A NUMBER**

Given: Table 2, Common Logarithms of Numbers.

Find: The logarithm of 1.64.

Solution: Locate the digits 16 in the left-hand column of Table 2 as shown below. Move horizontally to the right until the column headed by the 4 is intersected. The digits at the intersection are 2148, which represent the decimal fraction less than 1 and the logarithm of 1.64.

In summary:

$$\log (1.64) = 0.2148$$

Portion of Table 2

PORTION OF TABLE 2.

| N | 0 | 1 | 2 | 3 | (4) | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 0000 | 0043 | 0086 | 0128 | 0170 | 0212 | 0253 | 0294 | 0334 | 0374 |
| 11 | 0414 | 0453 | 0492 | 0531 | 0569 | 0607 | 0645 | 0682 | 0719 | 0755 |
| 12 | 0792 | 0828 | 0864 | 0899 | 0934 | 0969 | 1004 | 1038 | 1072 | 1106 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 |
| 14 | 1461 | 1492 | 1523 | 1553 | 1584 | 1614 | 1644 | 1673 | 1703 | 1732 |
| 15 | 1761 | 1790 | 1818 | 1847 | 1875 | 1903 | 1931 | 1959 | 1987 | 2014 |
| 16 | | | | | (2148) | 2175 | 2201 | 2227 | 2253 | 2279 |
| 17 | 2304 | 2330 | 2355 | 2380 | 2405 | 2430 | 2455 | 2480 | 2504 | 2529 |
| 18 | 2553 | 2577 | 2601 | 2625 | 2648 | 2672 | 2695 | 2718 | 2742 | 2765 |
| 19 | 2788 | 2810 | 2833 | 2856 | 2878 | 2900 | 2923 | 2945 | 2967 | 2989 |

To find logarithms of numbers greater than 10, the numbers must be converted to scientific notation, and use must be made of a property of logarithms that states that "the logarithm of the product of two numbers is equal to the sum of the logarithms of the individual numbers." For instance, the logarithm of 16.4 is found when 16.4 is converted to $1.64 \times 10^1$, as witnessed in Example B.

**EXAMPLE B: FINDING THE LOGARITHM OF A NUMBER GREATER THAN TEN**

Given: Table 2, Common Logarithms of Numbers.

Find: Log (16.4).

Solution: $\log (16.4) = \log (1.64 \times 10^1) = \log (1.64) + \log (10^1)$

$= 0.2148 + 1.000$

$\log (16.4) = 1.2148$

which means that, $10^{1.2148} = 16.4$ in exponential form

Likewise,

$\log (164.) = \log (1.64 \times 10^2) = \log (1.64) + \log (10^2)$

$= 0.2148 + 2$

$\log (164) = 2.2148$

or equivalently, $(10^{2.2148} = 164)$

The logarithm of the power-of-ten is called the "characteristic," and the logarithm of the first factor of a number containing the significant digits in scientific notation is called the "mantissa." The mantissa is the logarithm located in the table of common logarithms; therefore, the logarithm of any number greater than 10 is the sum of a mantissa and a characteristic.

Negative logarithms frequently cause difficulties. Recall that power-of-ten numbers less than 1 have negative exponents ($0.01 = 10^{-2}$). To determine logarithms of numbers between 0 and 1, write the number in standard scientific notation; find its mantissa in the log table; and supply its negative characteristic. This procedure is clarified in Example C.

**EXAMPLE C: FINDING THE LOGARITHM OF A NUMBER LESS THAN ONE**

Given: Table 2, Common Logarithms of Numbers.

Find: The logarithm of 0.0265.

Solution: (1) Write 0.0265 as $2.65 \times 10^{-2}$.

(2) Find the mantissa in the log table. Its mantissa is 0.4232.

(3) Its characteristic is -2 (a negative exponent).

Therefore: $\log (0.0265) = 0.4232 + (-2)$

$= -1.5768$

## FINDING THE ANTILOGARITHM

The number corresponding to a given logarithm often must be found. As an example, find x, given that $\log x = 0.5999$. Since the characteristic is 0, x must be a number between 1 and 10. The digits 5999 in Table 2 are located in such a manner so as to corres-

pond to 39 in the left-hand column (laterally) and to the number 8 heading in the vertical column; therefore, $x = 3.98$ ($\log (3.98) = 0.5999$).

If y is the logarithm of a number x, then x is called the "antilog" of y, as in Example D.

**EXAMPLE D: FINDING THE ANTILOG OF A NUMBER**

Given: Table 2, Common Logarithms of Numbers.

Find: (a) The antilog of 2.9547.
(b) The antilog of -4.3655.

Solution: (a) From the table of common logarithms, the antilog

$0.9547 = 9.01.$

Then the antilog

$2.9547 = 9.01 \times 10^{2}$

$= 901.$

(b) Antilog $(-4.3655)$ = Antilog $(-5 + 0.6345)$.

From the table of common logarithms the antilog

$0.6345 = 4.31.$

Then, the antilog of

$(-5 + 0.6345) = 4.31 \times 10^{-5}$

$= 0.0000431.$

The reasoning for the first step in Example D is the fact that mantissas always must be positive numbers, although characteristics may be negative or positive. If -4.3655 had been represented as (-4-.3655), then -.3655 could not be the mantissa, because it is not a positive number as required.

## INTERPOLATION

To find the logarithm of a number that lies between two given numbers in Table 2, one can assume a linear change in numbers between the nearest logarithms and interpolate, as shown in Example E.

**EXAMPLE E: INTERPOLATION**

Given: Table 2, Common Logarithms of Numbers.

Find: log 247.2.

Solution:

$$\begin{aligned} \log 248 &= 2.3945 \\ \text{and} \quad \log 247 &= 2.3927 \\ \text{difference} &= 0.0018 \end{aligned}$$

$$\begin{aligned} \text{Then, } \log 247.2 &= 2.3927 + (0.2) \times (0.0018) \\ &= 2.3927 + .0003' \\ \log 247.2 &= 2.39306 \approx 2.3931 \end{aligned}$$

## EVALUATING FORMULAS CONTAINING LOGARITHMS

The preceding discussion of the mechanics of obtaining logarithms is a necessary prerequisite to any evaluation of formulas containing logarithms. One such formula is the equation for gain used in the study of electrical power, light energy, and sound energy. Its form is given below:

$$D = 10 \log \left(\frac{P_1}{P_2}\right)$$

In this formula, $\log (P_2/P_1)$ is the logarithm of a fraction. If the values of $P_2$ and $P_1$ are known, the corresponding value of D can be found as shown in Example F.

**EXAMPLE F: SOLVING LOGARITHMIC EQUATIONS**

Given: $D = 10 \log (P_2/P_1)$.

$P_2 = 15.5$, and $P_1 = 12.7$.

Find: D.

Solution:

$$D = 10 \log \left(\frac{15.5}{12.7}\right)$$

$$D = 10 \log (1.22)$$

Use the log tables:

$$D = 10 \times (0.0864)$$

$$D = 0.864$$

Frequently, the antilogarithm must be obtained as shown in Example G.

**EXAMPLE G: SOLVING LOGARITHMIC EQUATIONS**

Given: $D = 10 \log (P_2/P_1)$.

$D = 18.7$.

Find: The ratio of $P_2/P_1$.

Solution:

$$\log \left(\frac{P_2}{P_1}\right) = \frac{D}{10}$$

$$\log \left(\frac{P_2}{P_1}\right) = \frac{18.7}{10}$$

$$\log \left(\frac{P_2}{P_1}\right) = 1.87$$

Find the antilog:

$$(P_2/P_1) = 74.1$$

## NATURAL LOGARITHMS

Although numbers can be expressed as logarithms (exponents) to any base, only two bases are in general use — (1) the number 10 (common logarithms) and (2) the number e (2.7182818 ....). The symbol "e" comes from the first letter of "exponential" and describes any changing process whose rate of change is proportional to its instantaneous magnitude. Such processes include growth and decay, absorption, capacitor charge, and inductive reactions. The formulas describing these processes contain the symbol e as a base with some exponent. The following are some example equations:

$$q = Q\,(1 - 3^{-kt}) \quad \text{and} \quad y = A\,e^{+kx}.$$

Just as any positive number can be written in a power-of-ten form, any positive number can be written in power of "e" form. Since e is approximately 2.72,

$$2.72 = 10^{0.4346}, \text{ or}$$

more accurately, $e = 10^{0.4343}$.

## TABLE 2. COMMON LOGARITHMS OF NUMBERS

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 0000 | 0043 | 0086 | 0128 | 0170 | 0212 | 0253 | 0294 | 0334 | 0374 |
| 11 | 0414 | 0453 | 0492 | 0531 | 0569 | 0607 | 0645 | 0682 | 0719 | 0755 |
| 12 | 0792 | 0828 | 0864 | 0899 | 0934 | 0969 | 1004 | 1038 | 1072 | 1106 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 |
| 14 | 1461 | 1492 | 1523 | 1553 | 1584 | 1614 | 1644 | 1673 | 1703 | 1732 |
| 15 | 1761 | 1790 | 1818 | 1847 | 1875 | 1903 | 1931 | 1959 | 1987 | 2014 |
| 16 | 2041 | 2068 | 2095 | 2122 | 2148 | 2175 | 2201 | 2227 | 2253 | 2279 |
| 17 | 2304 | 2330 | 2355 | 2380 | 2405 | 2430 | 2455 | 2480 | 2504 | 2529 |
| 18 | 2553 | 2577 | 2601 | 2625 | 2648 | 2672 | 2695 | 2718 | 2742 | 2765 |
| 19 | 2788 | 2810 | 2833 | 2856 | 2878 | 2900 | 2923 | 2945 | 2967 | 2989 |
| 20 | 3010 | 3032 | 3054 | 3075 | 3096 | 3118 | 3139 | 3160 | 3181 | 3201 |
| 21 | 3222 | 3243 | 3263 | 3284 | 3304 | 3324 | 3345 | 3365 | 3385 | 3404 |
| 22 | 3424 | 3444 | 3464 | 3483 | 3502 | 3522 | 3541 | 3560 | 3579 | 3598 |
| 23 | 3617 | 3636 | 3655 | 3674 | 3692 | 3711 | 3729 | 3747 | 3766 | 3784 |
| 24 | 3802 | 3820 | 3838 | 3856 | 3874 | 3892 | 3909 | 3927 | 3945 | 3962 |
| 25 | 3979 | 3997 | 4014 | 4031 | 4048 | 4065 | 4082 | 4099 | 4116 | 4133 |
| 26 | 4150 | 4166 | 4183 | 4200 | 4216 | 4232 | 4249 | 4265 | 4281 | 4298 |
| 27 | 4314 | 4330 | 4346 | 4362 | 4378 | 4393 | 4409 | 4425 | 4440 | 4456 |
| 28 | 4472 | 4487 | 4502 | 4518 | 4533 | 4548 | 4564 | 4579 | 4594 | 4609 |
| 29 | 4624 | 4639 | 4654 | 4669 | 4683 | 4698 | 4713 | 4728 | 4742 | 4757 |
| 30 | 4771 | 4786 | 4800 | 4814 | 4829 | 4843 | 4857 | 4871 | 4886 | 4900 |
| 31 | 4914 | 4928 | 4942 | 4955 | 4969 | 4983 | 4997 | 5011 | 5024 | 5038 |
| 32 | 5051 | 5065 | 5079 | 5092 | 5105 | 5119 | 5132 | 5145 | 5159 | 5172 |
| 33 | 5185 | 5198 | 5211 | 5224 | 5237 | 5250 | 5263 | 5276 | 5289 | 5302 |
| 34 | 5315 | 5328 | 5340 | 5353 | 5366 | 5378 | 5391 | 5403 | 5416 | 5428 |
| 35 | 5441 | 5453 | 5465 | 5478 | 5490 | 5502 | 5514 | 5527 | 5539 | 5551 |
| 36 | 5563 | 5575 | 5587 | 5599 | 5611 | 5623 | 5635 | 5647 | 5658 | 5670 |
| 37 | 5682 | 5694 | 5705 | 5717 | 5729 | 5740 | 5752 | 5763 | 5775 | 5786 |
| 38 | 5798 | 5809 | 5821 | 5832 | 5843 | 5855 | 5866 | 5877 | 5888 | 5899 |
| 39 | 5911 | 5922 | 5933 | 5944 | 5955 | 5966 | 5977 | 5988 | 5999 | 6010 |
| 40 | 6021 | 6031 | 6042 | 6053 | 6064 | 6075 | 6085 | 6096 | 6107 | 6117 |
| 41 | 6128 | 6138 | 6149 | 6160 | 6170 | 6180 | 6191 | 6201 | 6212 | 6222 |
| 42 | 6232 | 6243 | 6253 | 6263 | 6274 | 6284 | 6294 | 6304 | 6314 | 6325 |
| 43 | 6335 | 6345 | 6355 | 6365 | 6375 | 6385 | 6395 | 6405 | 6415 | 6425 |
| 44 | 6435 | 6444 | 6454 | 6464 | 6474 | 6484 | 6493 | 6503 | 6513 | 6522 |
| 45 | 6532 | 6542 | 6551 | 6561 | 6571 | 6580 | 6590 | 6599 | 6609 | 6618 |
| 46 | 6628 | 6637 | 6646 | 6656 | 6665 | 6675 | 6684 | 6693 | 6702 | 6712 |
| 47 | 6721 | 6730 | 6739 | 6749 | 6758 | 6767 | 6776 | 6785 | 6794 | 6803 |
| 48 | 6812 | 6821 | 6830 | 6839 | 6848 | 6857 | 6866 | 6875 | 6884 | 6893 |
| 49 | 6902 | 6911 | 6920 | 6928 | 6937 | 6946 | 6955 | 6964 | 6972 | 6981 |
| 50 | 6990 | 6998 | 7007 | 7016 | 7024 | 7033 | 7042 | 7050 | 7059 | 7067 |
| 51 | 7076 | 7084 | 7093 | 7101 | 7110 | 7118 | 7126 | 7135 | 7143 | 7152 |
| 52 | 7160 | 7168 | 7177 | 7185 | 7193 | 7202 | 7210 | 7218 | 7226 | 7235 |
| 53 | 7243 | 7251 | 7259 | 7267 | 7275 | 7284 | 7292 | 7300 | 7308 | 7316 |
| 54 | 7324 | 7332 | 7340 | 7348 | 7356 | 7364 | 7372 | 7380 | 7388 | 7396 |

TABLE 2. - CONTINUED

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 65 | 7404 | 7412 | 7419 | 7427 | 7435 | 7443 | 7451 | 7459 | 7466 | 7474 |
| 66 | 7482 | 7490 | 7497 | 7505 | 7513 | 7520 | 7528 | 7536 | 7543 | 7551 |
| 67 | 7559 | 7566 | 7574 | 7582 | 7589 | 7597 | 7604 | 7612 | 7619 | 7627 |
| 68 | 7634 | 7642 | 7649 | 7657 | 7664 | 7672 | 7679 | 7686 | 7694 | 7701 |
| 69 | 7709 | 7716 | 7723 | 7731 | 7738 | 7745 | 7752 | 7760 | 7767 | 7774 |
| 60 | 7782 | 7789 | 7796 | 7803 | 7810 | 7818 | 7825 | 7832 | 7839 | 7846 |
| 61 | 7853 | 7860 | 7868 | 7875 | 7882 | 7889 | 7896 | 7903 | 7910 | 7917 |
| 62 | 7924 | 7931 | 7938 | 7945 | 7952 | 7959 | 7966 | 7973 | 7980 | 7987 |
| 63 | 7993 | 8000 | 8007 | 8014 | 8021 | 8028 | 8035 | 8041 | 8048 | 8055 |
| 64 | 8062 | 8069 | 8075 | 8082 | 8089 | 8096 | 8102 | 8109 | 8116 | 8122 |
| 66 | 8129 | 8136 | 8142 | 8149 | 8156 | 8162 | 8169 | 8176 | 8182 | 8189 |
| 66 | 8195 | 8202 | 8209 | 8215 | 8222 | 8228 | 8235 | 8241 | 8248 | 8254 |
| 67 | 8261 | 8267 | 8274 | 8280 | 8287 | 8293 | 8299 | 8306 | 8312 | 8319 |
| 68 | 8325 | 8331 | 8338 | 8344 | 8351 | 8357 | 8363 | 8370 | 8376 | 8382 |
| 69 | 8388 | 8395 | 8401 | 8407 | 8414 | 8420 | 8426 | 8432 | 8439 | 8445 |
| 70 | 8451 | 8457 | 8463 | 8470 | 8476 | 8482 | 8488 | 8494 | 8500 | 8506 |
| 71 | 8513 | 8519 | 8525 | 8531 | 8537 | 8543 | 8549 | 8555 | 8561 | 8567 |
| 72 | 8573 | 8579 | 8585 | 8591 | 8597 | 8603 | 8609 | 8615 | 8621 | 8627 |
| 73 | 8633 | 8639 | 8645 | 8651 | 8657 | 8663 | 8669 | 8675 | 8681 | 8686 |
| 74 | 8692 | 8698 | 8704 | 8710 | 8716 | 8722 | 8727 | 8733 | 8739 | 8745 |
| 75 | 8751 | 8756 | 8762 | 8768 | 8774 | 8779 | 8785 | 8791 | 8797 | 8802 |
| 76 | 8808 | 8814 | 8820 | 8825 | 8831 | 8837 | 8842 | 8848 | 8854 | 8859 |
| 77 | 8865 | 8871 | 8876 | 8882 | 8887 | 8893 | 8899 | 8904 | 8910 | 8915 |
| 78 | 8921 | 8927 | 8932 | 8938 | 8943 | 8949 | 8954 | 8960 | 8965 | 8971 |
| 79 | 8976 | 8982 | 8987 | 8993 | 8998 | 9004 | 9009 | 9015 | 9020 | 9025 |
| 80 | 9031 | 9036 | 9042 | 9047 | 9053 | 9058 | 9063 | 9069 | 9074 | 9079 |
| 81 | 9085 | 9090 | 9096 | 9101 | 9106 | 9112 | 9117 | 9122 | 9128 | 9133 |
| 82 | 9138 | 9143 | 9149 | 9154 | 9159 | 9165 | 9170 | 9175 | 9180 | 9186 |
| 83 | 9191 | 9196 | 9201 | 9206 | 9212 | 9217 | 9222 | 9227 | 9232 | 9238 |
| 84 | 9243 | 9248 | 9253 | 9258 | 9263 | 9269 | 9274 | 9279 | 9284 | 9289 |
| 85 | 9294 | 9299 | 9304 | 9309 | 9315 | 9320 | 9325 | 9330 | 9335 | 9340 |
| 86 | 9345 | 9350 | 9355 | 9360 | 9365 | 9370 | 9375 | 9380 | 9385 | 9390 |
| 87 | 9395 | 9400 | 9405 | 9410 | 9415 | 9420 | 9425 | 9430 | 9435 | 9440 |
| 88 | 9445 | 9450 | 9455 | 9460 | 9465 | 9469 | 9474 | 9479 | 9484 | 9489 |
| 89 | 9494 | 9499 | 9504 | 9509 | 9513 | 9518 | 9523 | 9528 | 9533 | 9538 |
| 90 | 9542 | 9547 | 9552 | 9557 | 9562 | 9566 | 9571 | 9576 | 9581 | 9586 |
| 91 | 9590 | 9595 | 9600 | 9605 | 9609 | 9614 | 9619 | 9624 | 9628 | 9633 |
| 92 | 9638 | 9643 | 9647 | 9652 | 9657 | 9661 | 9666 | 9671 | 9675 | 9680 |
| 93 | 9685 | 9689 | 9694 | 9699 | 9703 | 9708 | 9713 | 9717 | 9722 | 9727 |
| 94 | 9731 | 9736 | 9741 | 9745 | 9750 | 9754 | 9759 | 9763 | 9768 | 9773 |
| 95 | 9777 | 9782 | 9786 | 9791 | 9795 | 9800 | 9805 | 9809 | 9814 | 9818 |
| 96 | 9823 | 9827 | 9832 | 9836 | 9841 | 9845 | 9850 | 9854 | 9859 | 9863 |
| 97 | 9868 | 9872 | 9877 | 9881 | 9886 | 9894 | 9898 | 9899 | 9903 | 9908 |
| 98 | 9912 | 9917 | 9921 | 9926 | 9930 | 9934 | 9939 | 9943 | 9948 | 9952 |
| 99 | 9956 | 9961 | 9965 | 9969 | 9974 | 9978 | 9983 | 9987 | 9991 | 9996 |

The numerical value of any power-of-e can be found by the conversion of power-of-e to power-of-ten form. This conversion simply requires the use of the common logarithm table, as shown in Example H.

**EXAMPLE H: FINDING $e^x$**

Given: Table 2, Common Logarithms of Numbers.

Find: $e^5$.

Solution: $e^5 = (10^{0.4343})^5 = 10^{(0.4343) \times 5}$

$e^5 = 10^{2.1715}$

Use Table 2 of Common Logarithms as shown here:

COMMON LOGARITHMS OF NUMBERS

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 0000 | 0043 | 0086 | 0128 | 0170 | 0212 | 0253 | 0294 | 0334 | 0374 |
| 11 | 0414 | 0453 | 0492 | 0531 | 0569 | 0607 | 0645 | 0682 | 0719 | 0755 |
| 12 | 0792 | 0828 | 0864 | 0899 | 0934 | 0969 | 1004 | 1038 | 1072 | 1106 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 |
| 14 | 1461 | 1492 | 1523 | 1553 | 1584 | 1614 | 1644 | 1673 | 1703 | 1732 |

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By interpolation:

$\log (149) = 2.1732$

$\log (148.77) = 2.1715$

$\log (148) = 2.1701$

$e^5 = 10^{2.1715} = 148 + \left(\frac{0.0012}{0.0029}\right) \times 1$

$= 148 + 0.414$

Therefore, $e^5 = 10^{2.1715} = 148.414$

This method, however, is tedious and time consuming. Mathematicians have prepared a table of power of e to three significant digits. Table 3 of this module is a table of $e^x$ and $e^{-x}$. Locate the power in the x column and read $e^x$ or $e^{-x}$ under the respective column of Table 3:

$e^5 = 148$ (to three significant digits).

## TABLE 3. TABLE OF $e^x$ AND $e^{-x}$

| x | $e^x$ | $e^{-x}$ | x | $e^x$ | $e^{-x}$ | x | $e^x$ | $e^{-x}$ |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 1.00 | 1.00 | 3.3 | 27.1 | 0.0369 | 6.6 | 735. | 0.00136 |
| 0.1 | 1.11 | 0.905 | 3.4 | 30.0 | 0.0334 | 6.7 | 812. | 0.00123 |
| 0.2 | 1.22 | 0.819 | | | | 6.8 | 898. | 0.00111 |
| 0.3 | 1.35 | 0.741 | 3.5 | 33.1 | 0.0302 | 6.9 | 992. | 0.00101 |
| 0.4 | 1.49 | 0.670 | 3.6 | 36.6 | 0.0273 | | | |
| | | | 3.7 | 40.4 | 0.0247 | 7.0 | 1100 | 0.000912 |
| 0.5 | 1.65 | 0.607 | 3.8 | 44.7 | 0.0224 | 7.1 | 1210 | 0.000825 |
| 0.6 | 1.82 | 0.549 | 3.9 | 49.4 | 0.0202 | 7.2 | 1340 | 0.000747 |
| 0.7 | 2.01 | 0.497 | | | | 7.3 | 1480 | 0.000676 |
| 0.8 | 2.23 | 0.449 | 4.0 | 54.6 | 0.0183 | 7.4 | 1640 | 0.000611 |
| 0.9 | 2.46 | 0.407 | 4.1 | 60.3 | 0.0166 | | | |
| | | | 4.2 | 66.7 | 0.0150 | 7.5 | 1810 | 0.000553 |
| 1.0 | 2.72 | 0.368 | 4.3 | 73.7 | 0.0136 | 7.6 | 2000 | 0.000500 |
| 1.1 | 3.00 | 0.333 | 4.4 | 81.5 | 0.0123 | 7.7 | 2210 | 0.000453 |
| 1.2 | 3.32 | 0.301 | | | | 7.8 | 2440 | 0.000410 |
| 1.3 | 3.67 | 0.273 | 4.5 | 90.0 | 0.0111 | 7.9 | 2700 | 0.000371 |
| 1.4 | 4.06 | 0.247 | 4.6 | 99.5 | 0.0101 | | | |
| | | | 4.7 | 110. | 0.00910 | 8.0 | 2980 | 0.000336 |
| 1.5 | 4.48 | 0.223 | 4.8 | 122. | 0.00823 | 8.1 | 3290 | 0.000304 |
| 1.6 | 4.95 | 0.202 | 4.9 | 134. | 0.00745 | 8.2 | 3640 | 0.000275 |
| 1.7 | 5.47 | 0.183 | | | | 8.3 | 4020 | 0.000248 |
| 1.8 | 6.05 | 0.165 | 5.0 | 148. | 0.00674 | 8.4 | 4450 | 0.000225 |
| 1.9 | 6.69 | 0.150 | 5.1 | 164. | 0.00610 | | | |
| | | | 5.2 | 181. | 0.00552 | 8.5 | 4910 | 0.000204 |
| 2.0 | 7.39 | 0.135 | 5.3 | 200. | 0.00499 | 8.6 | 5430 | 0.000184 |
| 2.1 | 8.17 | 0.122 | 5.4 | 221. | 0.00452 | 8.7 | 6000 | 0.000167 |
| 2.2 | 9.02 | 0.111 | | | | 8.8 | 6630 | 0.000151 |
| 2.3 | 9.97 | 0.100 | 5.5 | 245. | 0.00409 | 8.9 | 7330 | 0.000136 |
| 2.4 | 11.0 | 0.0907 | 5.6 | 270. | 0.00370 | | | |
| | | | 5.7 | 299. | 0.00335 | 9.0 | 8100 | 0.000123 |
| 2.5 | 12.2 | 0.0821 | 5.8 | 330. | 0.00303 | 9.1 | 8960 | 0.000112 |
| 2.6 | 13.5 | 0.0743 | 5.9 | 365. | 0.00274 | 9.2 | 9900 | 0.000101 |
| 2.7 | 14.9 | 0.0672 | | | | 9.3 | 10900 | 0.0000914 |
| 2.8 | 16.4 | 0.0608 | 6.0 | 403. | 0.00248 | 9.4 | 12100 | 0.0000827 |
| 2.9 | 18.2 | 0.0550 | 6.1 | 446. | 0.00224 | | | |
| | | | 6.2 | 493. | 0.00203 | 9.5 | 13400 | 0.0000749 |
| 3.0 | 20.1 | 0.0498 | 6.3 | 545. | 0.00184 | 9.6 | 14800 | 0.0000677 |
| 3.1 | 22.2 | 0.0450 | 6.4 | 602. | 0.00166 | 9.7 | 16300 | 0.0000613 |
| 3.2 | 24.5 | 0.0408 | | | | 9.8 | 18000 | 0.0000555 |
| | | | 6.5 | 665. | 0.00150 | 9.9 | 19900 | 0.0000502 |
| | | | | | | 10.0 | 22000 | 0.0000454 |

Natural logarithms are based upon the power of "e." The natural logarithm of a number is simply the exponent when it is written in "power of e" form. Any power of e can be written in logarithmic notation. The symbol "ln" is used when reference is made to natural logarithms. Notice, for example, these forms:

$$602 = e^{6.4} \quad \text{(power-of-e form)}$$

$$\ln 602 = 6.4 \quad \text{(logarithmic equivalent).}$$

Mathematicians have prepared tables of natural logarithms, and these tables can be found in many handbooks. Table 4 of this module entitled, "Table of Natural Logarithms," is such a representative table. Examine the table, and note that "N" stands for the number and that "ln N" stands for the natural logarithm. Unlike the common log table, the natural log table provides the entire logarithm, as in these examples:

$$\ln (1.60) = 0.4700$$

$$\ln (45.0) = 3.8067$$

Logarithms determined by tables are approximate and are correct to as many significant figures as the number of places in the table used. A table that gives the desired number of significant figures should be used.

## APPLICATIONS OF NATURAL LOGARITHMS

A process in which a rate of change of a variable decreases as time proceeds is known as a "decay process." Such a process occurs as an object cools down, as a capacitor discharges, or as radioactive material decays. Such a process is illustrated graphically in Figure 4. The curve is described mathematically by Equation 1.

$$P = P_o e^{-kt} \qquad (1)$$

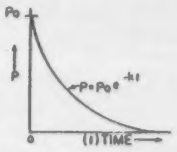


FIG. 4

where

$P_o$ = initial value of the decaying quantity.
$t$ = time.
$k$ = a constant factor controlling the rapidity of the decay (the larger the k value, the more rapid the decay process).

## TABLE 4. NATURAL LOGARITHMS

| N | ln N | N | ln N | N | ln N | N | ln N |
|---|---|---|---|---|---|---|---|
| 0.00 | ------- | 5.00 | 1.6094 | 10.0 | 2.3026 | 60.0 | 4.0943 |
| 0.10 | -2.3026 | 5.10 | 1.6292 | 11.0 | 2.3979 | 61.0 | 4.1109 |
| 0.20 | -1.6094 | 5.20 | 1.6487 | 12.0 | 2.4849 | 62.0 | 4.1271 |
| 0.30 | -1.2040 | 5.30 | 1.6677 | 13.0 | 2.5650 | 63.0 | 4.1431 |
| 0.40 | -0.9163 | 5.40 | 1.6864 | 14.0 | 2.6391 | 64.0 | 4.1589 |
| 0.50 | -0.6932 | 5.50 | 1.7048 | 15.0 | 2.7080 | 65.0 | 4.1744 |
| 0.60 | -0.5108 | 5.60 | 1.7228 | 16.0 | 2.7726 | 66.0 | 4.1896 |
| 0.70 | -0.3567 | 5.70 | 1.7405 | 17.0 | 2.8332 | 67.0 | 4.2047 |
| 0.80 | -0.2231 | 5.80 | 1.7579 | 18.0 | 2.8904 | 68.0 | 4.2195 |
| 0.90 | -0.1054 | 5.90 | 1.7750 | 19.0 | 2.9444 | 69.0 | 4.2341 |
| 1.00 | 0.0000 | 6.00 | 1.7918 | 20.0 | 2.9957 | 70.0 | 4.2485 |
| 1.10 | 0.0953 | 6.10 | 1.8083 | 21.0 | 3.0445 | 71.0 | 4.2627 |
| 1.20 | 0.1823 | 6.20 | 1.8246 | 22.0 | 3.0910 | 72.0 | 4.2767 |
| 1.30 | 0.2624 | 6.30 | 1.8406 | 23.0 | 3.1355 | 73.0 | 4.2905 |
| 1.40 | 0.3365 | 6.40 | 1.8563 | 24.0 | 3.1780 | 74.0 | 4.3041 |
| 1.50 | 0.4055 | 6.50 | 1.8718 | 25.0 | 3.2189 | 75.0 | 4.3175 |
| 1.60 | 0.4700 | 6.60 | 1.8871 | 26.0 | 3.2581 | 76.0 | 4.3307 |
| 1.70 | 0.5306 | 6.70 | 1.9021 | 27.0 | 3.2958 | 77.0 | 4.3438 |
| 1.80 | 0.5878 | 6.80 | 1.9169 | 28.0 | 3.3322 | 78.0 | 4.3567 |
| 1.90 | 0.6418 | 6.90 | 1.9315 | 29.0 | 3.3673 | 79.0 | 4.3694 |
| 2.00 | 0.6932 | 7.00 | 1.9459 | 30.0 | 3.4012 | 80.0 | 4.3820 |
| 2.10 | 0.7419 | 7.10 | 1.9601 | 31.0 | 3.4340 | 81.0 | 4.3944 |
| 2.20 | 0.7885 | 7.20 | 1.9741 | 32.0 | 3.4657 | 82.0 | 4.4067 |
| 2.30 | 0.8329 | 7.30 | 1.9879 | 33.0 | 3.4965 | 83.0 | 4.4188 |
| 2.40 | 0.8755 | 7.40 | 2.0015 | 34.0 | 3.5264 | 84.0 | 4.4308 |
| 2.50 | 0.9163 | 7.50 | 2.0149 | 35.0 | 3.5554 | 85.0 | 4.4426 |
| 2.60 | 0.9555 | 7.60 | 2.0282 | 36.0 | 3.5835 | 86.0 | 4.4543 |
| 2.70 | 0.9932 | 7.70 | 2.0412 | 37.0 | 3.6109 | 87.0 | 4.4659 |
| 2.80 | 1.0296 | 7.80 | 2.0541 | 38.0 | 3.6376 | 88.0 | 4.4773 |
| 2.90 | 1.0647 | 7.90 | 2.0669 | 39.0 | 3.6636 | 89.0 | 4.4886 |
| 3.00 | 1.0986 | 8.00 | 2.0794 | 40.0 | 3.6889 | 90.0 | 4.4998 |
| 3.10 | 1.1314 | 8.10 | 2.0919 | 41.0 | 3.7136 | 91.0 | 4.5109 |
| 3.20 | 1.1632 | 8.20 | 2.1041 | 42.0 | 3.7377 | 92.0 | 4.5218 |
| 3.30 | 1.1939 | 8.30 | 2.1163 | 43.0 | 3.7612 | 93.0 | 4.5326 |
| 3.40 | 1.2238 | 8.40 | 2.1282 | 44.0 | 3.7842 | 94.0 | 4.5433 |
| 3.50 | 1.2528 | 8.50 | 2.1401 | 45.0 | 3.8067 | 95.0 | 4.5539 |
| 3.60 | 1.2809 | 8.60 | 2.1518 | 46.0 | 3.8286 | 96.0 | 4.5644 |
| 3.70 | 1.3083 | 8.70 | 2.1633 | 47.0 | 3.8502 | 97.0 | 4.5747 |
| 3.80 | 1.3350 | 8.80 | 2.1748 | 48.0 | 3.8712 | 98.0 | 4.5850 |
| 3.90 | 1.3610 | 8.90 | 2.1860 | 49.0 | 3.8918 | 99.0 | 4.5951 |
| 4.00 | 1.3863 | 9.00 | 2.1972 | 50.0 | 3.9120 | 100. | 4.6052 |
| 4.10 | 1.4110 | 9.10 | 2.2083 | 51.0 | 3.9318 | 150. | 5.0106 |
| 4.20 | 1.4351 | 9.20 | 2.2192 | 52.0 | 3.9512 | 200. | 5.2983 |
| 4.30 | 1.4586 | 9.30 | 2.2300 | 53.0 | 3.9703 | 250. | 5.5215 |
| 4.40 | 1.4816 | 9.40 | 2.2407 | 54.0 | 3.9890 | 300. | 5.7038 |
| 4.50 | 1.5041 | 9.50 | 2.2513 | 55.0 | 4.0073 | 350. | 5.8579 |
| 4.60 | 1.5261 | 9.60 | 2.2618 | 56.0 | 4.0254 | 400. | 5.9915 |
| 4.70 | 1.5476 | 9.70 | 2.2721 | 57.0 | 4.0430 | 450. | 6.1092 |
| 4.80 | 1.5686 | 9.80 | 2.2824 | 58.0 | 4.0604 | 500. | 6.2146 |
| 4.90 | 1.5892 | 9.90 | 2.2925 | 59.0 | 4.0775 | | |

In order to evaluate Equation 1 for the value of P — given $P_o = 40$, $k = 0.02$ and $t = 140$ — substitute the known values as follows:

$$P = (40)\ e^{-(0.02)(140)}$$

$$P = (40)\ e^{-(2.8)}$$

Use Table 3:

$$P = 40\ (0.0608)$$

$$P = 2.43.$$

Equation 1 can be rearranged to solve for t; for example, given the values $P_o = 40$, $k = 0.02$, and $P = 2.43$, t can be solved thusly:

| | Step | Explanation |
|---|---|---|
| Solution: | $P = P_o\ e^{-kt}$ | Write equation. |
| | $P\ e^{kt} = P_o\ \underbrace{(e^{-kt})(e^{kt})}_{=1}$ | Multiply by $e^{+kt}$. |
| | $e^{kt} = \frac{P_o}{P}$ (1) | Divide by P. |
| | $kt = \ln (\frac{P_o}{P})$ | Write equivalent logarithmic form. |
| | $t = \frac{1}{k} \ln (\frac{P_o}{P})$ | Divide by k. |
| | $t = \frac{\ln (40/2.43)}{0.02}$ | Substitute. |
| | $t = \frac{\ln (16.46)}{0.02} = \frac{2.80}{0.02}$ | |
| | $t = 140$ | |

Figure 5 illustrates a process that begins with a zero value of a quantity and grows exponentially as time increases toward some maximum value Q. The curve is described mathematically by Equation 2.

$$q = Q\ (1-e^{-kt}) \qquad (2)$$

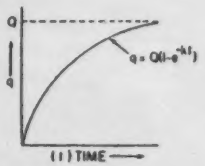


FIG. 5

An adiabatic expansion of a quantity of an ideal gas is described mathematically by Equation 3.

$$\text{Constant} = p\ V^n \qquad (3)$$

where

p = gas pressure.
V = initial volume of the gas.
n = exponent on V.

Logarithms are used, as in Example 1, to evaluate Equation 3 for the value of the Constant when $p$, $V$, and $n$ are given.

**EXAMPLE 5 FINDING THE CONSTANT**

**Given:** An ideal gas in a container exhibits a pressure of 600 and a volume of 140. The exponent has a value of 1.4. (Units are not considered here, in order that the mathematics itself will be clearer.)

**Find:** Value of constant.

**Solution:**

| Steps | Explanation |
|---|---|
| $\text{Constant} = (600)(140)^{1.4}$ | Susbtitute given data. |
| $\log(\text{Constant}) = \log[(600)(140)^{1.4}]$ | Write the logarithmic form. |
| | Apply a basic theorem of logarithms: $\log(x)(y) = \log x + \log y.$ |
| $\log(\text{Constant}) = \log(600) + \log[(140)^{1.4}]$ | |
| | Apply another basic theorem of logarithms: $\log x^n = n \log x.$ |
| $\log(\text{Constant}) = \log(600) + 1.4 \log(140)$ | |
| $= 2.7782 + 1.4 \times 2.1461$ | |
| $= 2.7782 + 3.0045$ | |
| $\log(\text{Constant}) = 5.7827$ | Find antilog. |
| $\text{Constant} = 6.063 \times 10^5$ | |
| or 606300 | (to four significant digits) |

In summary, three basic theorems aid in the performance of calculations that evaluate logarithms:

$$\log(x)(y) = \log x + \log y.$$
$$\log(x/y) = \log x - \log y.$$
$$\log(x^y) = y \log x.$$

Similar theorems hold also for natural logarithms.

The ability to use mathematical equations that include exponential and logarithmic terms is a necessary prerequisiste to the study of time constants (the subject of Concept Moudle I2-0). Work with logarithms and exponential notation should be done with extreme care. Remeber the ease with which one can "trip over logs." On the other hand, properly-handled exponential notation can be as easy as "falling off a log."

## SUMMARY

Exponential, or power-of-ten, notation facilitates the recording and manipulation of data by allowing large numbers to be written in abbreviated form.

Logarithms of numbers between 1.00 and 9.99 are called "common" (Table 2). A common logarithm is the exponential expression of a number having ten as the base; that is, the power-of-ten required to produce that number. Logs of numbers greater than 10 are found by conversion to scientific notation and by use of the following rule: "The logarithm of the product of two numbers is equal to the sum of the logarithms of the individual numbers." Often, the antilogarithm, or the number corresponding to a given logarithm, must be found. "Interpolation" is the practice of finding logs of numbers that lie between other numbers. "Natural logarithms" are employed to determine the rate of change in a process; they are valuable in industry.

## DETERMINATION OF LOGARITHMS AND USE OF EXPONENTS

### MATERIALS

Logarithm Tables in this module.
Scientific calculator.

### PROCEDURE

1. Express the following numbers in scientific notation:
   a. 43,762 = _____
   b. 29.78 = _____
   c. 0.013 = _____
   d. 0.000287 = _____

2. Evaluate the following, and express the results in scientific notation. Check your answers with your calculator.
   a. $2.97 \times 10^4 + 3.6 \times 10^1$ = _____
   b. $6.30 \times 10^3 - 6.30 \times 10^2$ = _____
   c. $(1.15 \times 10^{-2}) \times (3.0 \times 10^4)$ = _____
   d. $(4.25 \times 10^6) \div (2.5 \times 10^{-4})$ = _____

3. Determine the logarithms of the following numbers from the tables in this module. Interpolate if necessary. Check your answers with your calculator.
   a. log 235 = _____
   b. log 0.002 = _____
   c. ln 0.065 = _____
   d. ln 16.28 = _____

4. Use the three theorems of logarithms and the tables given in this module to evaluate the following. Check your answers with your calculator.
   a. $18.6 \times 5.2$ = _____
   b. $5.0 \div 27$ = _____
   c. $(21.92)^4$ = _____
   d. $(0.025) \times (8)^2 \div (14)^2$ = _____

5. If $D = 10 \log (P_2/P_1)$, find the following:

   a. D when $P_2 = 4$, $P_1 = 3$ $\quad D =$ _____

   b. $P_2$ when $P_1 = 6$, $D = 20$ $\quad P_2 =$ _____

   c. $P_1$ when $P_2 = 0.25$, $D = 2.5$ $\quad P_1 =$ _____

   d. D when $P_2 = 1$, $P_1 = 15$ $\quad D =$ _____

## SELF CHECK

1. Express the following numbers in scientific notation (power-of-ten):

   a. 2,930 = _____

   b. 262,000 = _____

   c. 704 = _____

   d. 0.9713 = _____

   e. 0.195 = _____

   f. 0.00000000833 = _____

2. Evaluate the following, and express the results in scientific notation:

   a. $6.61 \times 10^4 + 5.2 \times 10^1$ = _____

   b. $7.36 \times 10^{-6} + 2.74 \times 10^{-4}$ = _____

   c. $6.61 \times 10^3 - 4.41 \times 10^4$ = _____

   d. $(3.4 \times 10^{-3})(1.6 \times 10^{-4})$ = _____

   e. $(4.0 \times 10^{-3})(5.0 \times 10^3)$ = _____

   f. $\dfrac{6.4 \times 10^7}{2.5 \times 10^{-4}}$ = _____

3. Determine the logarithms of the following numbers, using the appropriate tables (interpolate if necessary):

   a. log 505 = _____

   b. log 0.001 = _____

   c. log 9, 28 = _____

   d. log 432.3 = _____

   e. ln 0.60 = _____

   f. ln 34.3 = _____

4. Using the thre theorems of logarithms and the log tables, evaluate the following:

   a. $21.65 \times 31.3$ = _____

   b. $1.0 \div 623$ = _____

   c. $(65.35)^3$ = _____

   d. $(65.2)(10.6)^2 \div 2.314$ = _____

5. In finding the insulation resistance by the leakage method, the resistance R is found from the formula —

$$R = 10^6 \times \frac{t}{c} \times \frac{1}{\ln \left(\frac{V_o}{V}\right)}$$

   where
   - t = time, seconds.
   - c = capacity.
   - V = voltage.

   Compute the value of R if $t = 120$, $V_o = 123$, $V = 115.8$, and $C = .082$.

6. In an alternating-current circuit, the current $i$ at any instant is given by the formula:

$$i = I \left(1 - e^{-\frac{Rt}{L}}\right)$$

   where
   - $i$ = instantaneous current.
   - I = maximum current.
   - R = resistance.
   - L = coefficient of self induction.
   - t = time.
   - e = base of natural logarithm.

   Evaluate $i$ when $I = 100$, $R = 16$, $L = 0.80$, and $t = 0.02$.

7. The voltage in a capacitor discharging is given by the equation

$$V = E\, e^{-\frac{t}{Rc}}$$

   Solve this equation for t, and evaluate the value of t, when $E = 200$, $t = 0.85$, $R = 12$, $c = 0.05$, $V = 49$.

8. The work done by a volume of gas, expanding at a constant temperature from a volume $V_o$ to a volume $V_1$, is given by the following formula:

$$W = p_o V_o \ln \left(\frac{V_1}{V_o}\right)$$

   Solve this equation for $V_1$.

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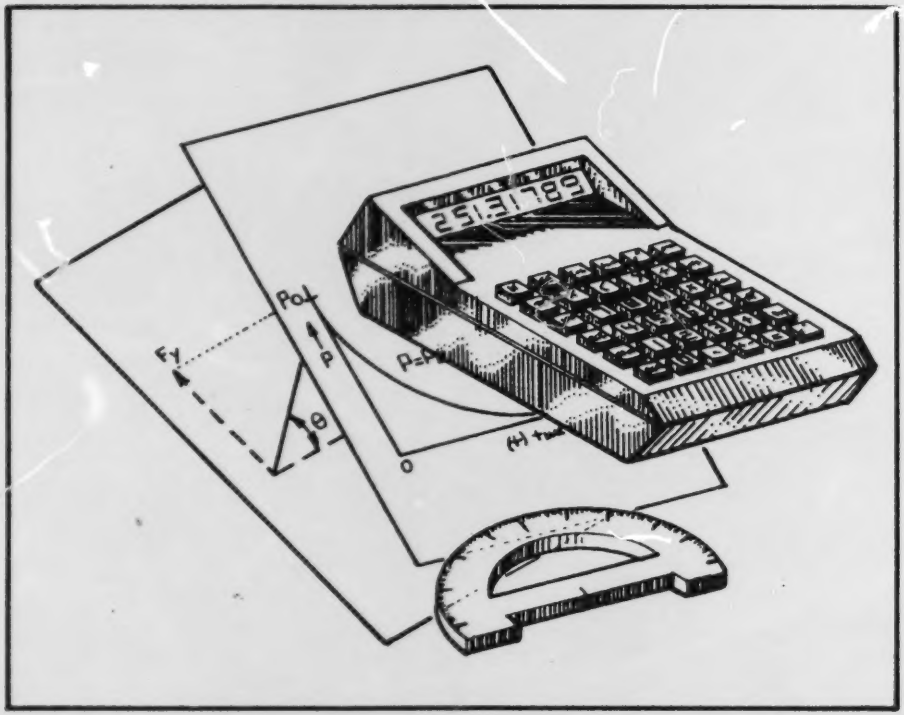
UNIFIED TECHNICAL CONCEPTS

MODULE S 4

ANGLES AND TRIANGLES




SUPPLEMENTARY

Technical Education Research Center-Southwest TERC-SW

## INTRODUCTION

The use of angles and triangles in technical work is very extensive and very important in preparation for Application Module IM2, "Addition of Mechanical Forces." This module introduces angles and their measurement to the extent necessary to understand their use in triangles of several different types. Similar triangles, right triangles, and the Pythagorean theorem are discussed. Trigonometric ratios of sine, cosine, and tangent are presented, as well as applications of these ratios to right angle, vector addition, and resolution of vectors into right-angle components. In the laboratory, the student will measure and/or calculate these quantities.

## PREREQUISITES

Student should have completed one year of high school algebra.

## OBJECTIVES

Upon completion of this module, the student should be able to:

- Identify the following terms:

  Angle.
  Right angle.
  Straight angle.
  Acute angle.
  Obtuse angle.
  Triangle.
  Right triangle.
  Acute triangle.
  Obtuse triangle
  Similar triangles.

- Measure an angle, given a protractor.

- Define these terms:

  Degree.
  Radian.
  Sine of an angle.
  Cosine of an angle.
  Tangent of an angle.

- Given two similar right triangles, a ruler, and a protractor, verify (a) that the ratio of corresponding sides of similar triangles are equal and (b) that all angles of a triangle add to 180°.

- Given a right triangle with two known wides, apply the Pythagorean theorem to determine the third side.

- Given stated problems involving right-triangle configurations, solve the problem for the unknown side or angle, using the definition of sine, cosine, or tangent, and the Pythagorean theorem.

- Given a vector, resolve it into two mutually-perpendicular component vectors.

## ANGLES (THE SUBJECT OF TRIGONOMETRY)

Two straight lines that meet at a point form an angle. Figure 1 shows the two straight lines AB and AC meeting at point A. Point A is called the "vertex of the angle." Often in diagrams involving angles, letters are placed near the vertex to identify the angle. Also the symbol ∠ often is used instead of the word "angle." The size of the angle is determined by the amount of rotation required if one side is rotated about the vertex into the other side, as shown in Figure 1. The measure of this rotation is expressed in a unit called a "degree." A degree is defined as 1/360 a circle; a circle includes 360 degrees. The symbol ° placed to the right of and above a number usually is employed instead of the word "degree." Sixty degrees is written 60°.

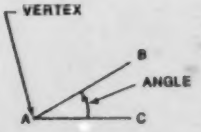



FIG. 1

Certain angles measured in degrees are given names. Figure 2 illustrates these angles. Two lines perpendicular to one another form a right angle of 90°; two lines forming a straight line make a straight angle of 180°. An angle less than 90° is called an "acute angle." An angle greater than 90° but less than 180° is termed an "obtuse angle."

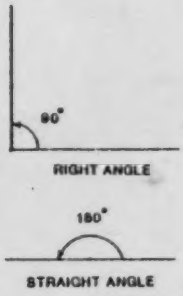



FIG. 2

### UNITS OF ANGULAR MEASURE

As mentioned previously, the degree is a common unit of angular measurement. However, it is a small unit of angular measurement. Figure 3 illustrates the size of one degree. In technical work, angles frequently are measured in a unit of larger size, called the "radian."

FIG. 3

The radian is defined in terms of a piece of a circle. A radian is the angle subtended by the arc of a circle equal in length to the radius. The angle θ produced at the center of the circle would be described as one radian. The angle θ would be equivalent to 57.2958 degrees. With an electronic calculator one can verify that one radian expressed in degrees multiplied by π(3.1415927) will give a result of 180 degrees. There are, therefore, π radians in a semicircle (180 degrees). This fact is often used in converting from degrees to radians or from radians to degrees, as indicated in Example A.

**EXAMPLE A: CONVERSIONS OF RADIANS AND DEGREES**

Given: Angles of 20° and 2.5 rad.

Find: (a) The equivalent of 20° in radians.
(b) The equivalent of 2.5 rad in degrees.

Solution:

| To convert — | multiply by — |
|---|---|
| degrees to radians | $\pi/180$ |
| radians to degrees | $180/\pi$ |

(a) $20° \times (\pi/180) = 0.35$ rad
(b) $2.5 \times (180/\pi) = 143°$

## MEASUREMENT OF ANGLES

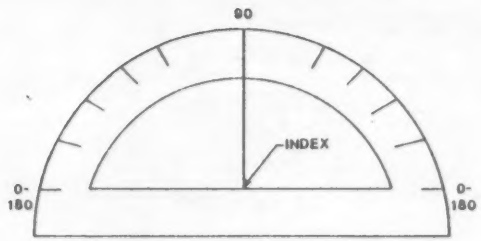



FIG. 4

The protractor, an instrument used in the measurement of angles, consists of a semicircular scale of convenient size (Figure 4). This scale is divided into degrees, and the divisions of the scale are numbered from 0° to 180°, beginning at each end. This method of numbering enables one to measure an angle from either end of the protractor.

To measure an angle with a protractor, align the protractor with one side of the angle to be measured so that the vertex of the angle and the index of the protractor coincide. (See Figure 5.) The reading on the scale where the second side of the angle crosses the scale part of the protractor is the measure of the angle in degrees. Figure 5 illustrates the measurement of a 25-degree angle.

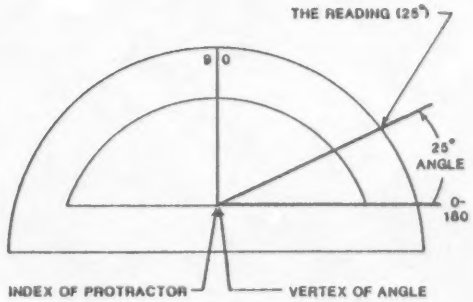



FIG. 5

## TRIANGLES

A polygon is a plane surface bounded by any number of straight lines called "sides." A triangle is a polygon of three sides, containing three angles, as shown in Figure 6. Applications of triangles are very common in industrial work such as bridge construction and architecture. Triangles often are classified by the

types of angles that they contain. Three of the angles used for classification were illustrated in Figure 2 of this module. An obtuse triangle contains one obtuse angle (greater than 90°) and two acute angles. A right triangle contains one right angle (equal to 90°) and two acute angles. An acute triangle contains only angles less than 90°. An example of each of these three types of triangles is given in Figure 7.

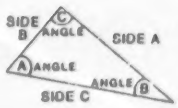
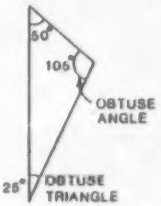



FIG. 6

One fundamental principle that applied to all triangles is the angle-sum principle:

THE SUM OF THE THREE ANGLES OF ANY TRIANGLE IS 180°

This fact can be vertified if the angles of each triangle illustrated in Figure 7 are added. The angle-sum principle is particularly useful when two angles of a triangle are known and the third angle is to be found, as in Example B.

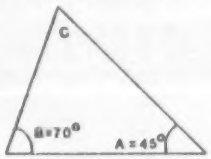



**EXAMPLE B: FINDING THE UNKNOWN ANGLE OF A TRIANGLE**

Given: A triangle contains angles of 70° and 45°.

Find: The third angle of the triangle.

Solution: The sum of the angles is 180°.

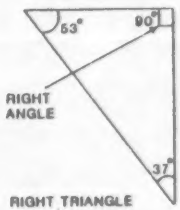



$\angle A + \angle B + \angle C = 180°$

$\angle C = 180° - A - B$

$\angle C = 180° - 70° - 45°$

$\angle C = 65°$




## SIMILAR TRIANGLES

When the angles of one triangle equal the angles of another triangle, the two triangles are said to be "similar." The triangles may be of different size, yet their corresponding angles are equal. Two similar triangles are illustrated in Figure 8. Corresponding sides of similar triangles are located across from the same angles of each triangle. In Figure 8, the 10-cm side in the large triangle and the 7.5-cm side in the smaller triangle are corresponding sides, both across the triangles from the 130° angle. Likewise, the 6.7-cm and 5.0-cm sides are corresponding, as are the 4.4-cm and 3.4-cm sides. Corresponding sides of similar triangles must have the same ratio. This ratio tells how much larger one triangle is than the other in terms of

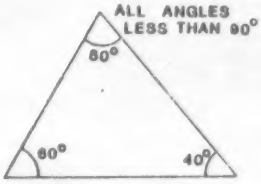
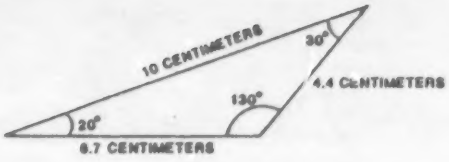



FIG. 7



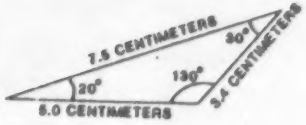




FIG. 8

its linear dimensions. For the two triangles in Figure 8, the ratios of corresponding sides can be computed as follows:

$$\frac{10\text{ cm}}{7.5\text{ cm}} = 1.3$$

$$\frac{6.7\text{ cm}}{5.0\text{ cm}} = 1.3$$

$$\frac{4.4\text{ cm}}{3.4\text{ cm}} = 1.3$$

These identical ratios show that the linear dimensions of the larger triangle are 1.3 times larger than the corresponding dimensions of the smaller triangle.

## RIGHT TRIANGLES

The most common use of ratios and proportions involving triangles is related to right triangles. A right triangle contains one 90° (right) angle. The side across the triangle from the 90° angle is called the "hypotenuse." The other two shorter sides sometimes are called "legs." A sample right triangle is shown in Figure 9. The relationship between the legs and hypotenuse of a right triangle is very important. This relationship is the well-known Pythagorean theorem: "The sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse." Figure 9 contains the approximate measurements of the legs and hypotenuse. These measurements can be verified by measurement with a centimeter scale. The use of the Pythagorean theorem to solve a problem is shown in Example C.

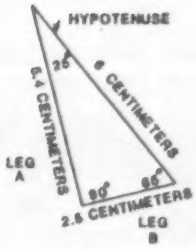



FIG. 9

**EXAMPLE C: PYTHAGOREAN THEOREM**

**Given:** The two legs of a right triangle are the same length as those in Figure 9.

**Find:** The hypotenuse.

**Solution:**

$(\text{hypotenuse})^2 = (\text{leg a})^2 + (\text{leg b})^2$ : The Pythagorean theorem

$(\text{hypotenuse})^2 = (5.4\text{ cm})^2 + (2.5\text{ cm})^2$

$(\text{hypotenuse})^2 = 29.16\text{ cm}^2 + 6.25\text{ cm}^2$

$(\text{hypotenuse})^2 = 35.41\text{ cm}^2$

Take the square root of both sides:

hypotenuse = 5.95 cm, or approximately 6 cm

## TRIGONOMETRIC RATIOS - SINE, COSINE, AND TANGENT

Certain ratios of lengths of sides of a right triangle are of fundamental importance in technical work. Before these important ratios can be explained, some new terminology associated with right triangles must be defined. In Figure 10, a right triangle is shown with one of the acute angles, $\theta$ (pronounced "thay-tuh"). The legs of the right triangle now are named in terms of their location relative to a particular angle of the right triangle. One side is opposite angle $\theta$ (across the triangle from $\theta$). The other leg is the side adjacent to angle $\theta$ (next to the angle $\theta$). The side adjacent to $\theta$ is one of the lines that defines the angle $\theta$. The hypotenuse, of course, is still the side across from the 90° angle. The ratios of interest are the following:

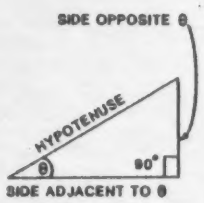



FIG. 10

- Length of side opposite $\theta$/length of hypotenuse.
- Length of side adjacent to $\theta$/length of hypotenuse.
- Length of side opposite $\theta$/length of side adjacent to $\theta$.

These ratios are constant for a given angle $\theta$ and are given the following names and abbreviations (abbr.):

- Sine $\theta = \dfrac{\text{length of side opposite } \theta}{\text{length of hypotenuse}}$ (abbr. sin $\theta$)
- Cosine $\theta = \dfrac{\text{length of side adjacent to } \theta}{\text{length of hypotenuse}}$ (abbr. cos $\theta$)
- Tangent $\theta = \dfrac{\text{length of side opposite to } \theta}{\text{length of side adjacent to } \theta}$ (abbr. tan $\theta$)

These names are read "sine of theta, cosine of theta, and tangent of theta." They should be committed to memory because of their extensive and frequent use in technology. Frequently, other Greek letters are used to label angles. These are some of the common letters used for angles:

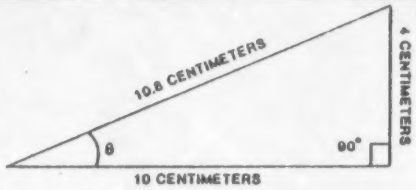
$\alpha$ (alpha) — pronounced "al-fuh."
$\beta$ (beta) — pronounced "bay-tuh."
$\gamma$ (gamma) — pronounced "gam-muh."
$\phi$ (phi) — pronounced "fee."

Example D demonstrates how to find the sine, cosine, and tangent ratios for a given right triangle.

**EXAMPLE D: SINE, COSINE, AND TANGENT RATIOS FOR A GIVEN ANGLE OF A RIGHT TRIANGLE**

**Given:** A right triangle has angles and sides with values shown in sketch.

**Find:** Sin θ, Cosine θ, Tangent θ.




**Solution:**

$$\text{Sin } \theta = \frac{\text{length of side opposite to } \theta}{\text{length of hypotenuse}} = \frac{4 \text{ cm}}{10.8 \text{ cm}}$$

$$\text{Sin } \theta = 0.370$$

$$\text{Cos } \theta = \frac{\text{length of side adjacent to } \theta}{\text{length of hypotenuse}} = \frac{10 \text{ cm}}{10.8 \text{ cm}}$$

$$\text{Cos } \theta = 0.925$$

$$\text{Tan } \theta = \frac{\text{length of side opposite to } \theta}{\text{length of side adjacent to } \theta} = \frac{4 \text{ cm}}{10 \text{ cm}}$$

$$\text{Tan } \theta = 0.400$$

The angle to which each ratio is related is approximately 22 degrees. This value can be verified by protactor measurements.

Note that in Example D, the values of the ratios are pure numbers and are directly related to the same angle. Any right triangle, regardless of size, containing the same 22-degree angle as in Example D would have the same values for the three trigonometric ratios of sine, cosine, and tangent.

The calculated ratios are useful if they are available for all acute angles. These ratios have been determined and arranged in tables. Table I illustrates the sine, cosine, and tangent ratio values for all acute angles from 0 to 90 degrees in one degree increments. The values of the sine and cosine are never greater than 1.000 or less than 0 for acute angles. The tangent value is greater than 1 for angles above 45°, and becomes extremely large for angles approaching 90°.

Many modern-day electronic calculators have sin, cos, and tan buttons that enable the user to obtain the trigonometric ratios. Usually, the angle is entered into the calculator keyboard in degrees; then the desired trigonometric ratio button is pushed ([sin], [cos], or [tan]). After a short time, the value of the desired ratio appears on the calculator display. The owner's manual that comes with most electronic calculators is helpful in determining the right sequence of operations.

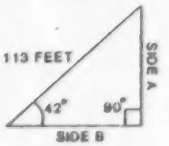
## TABLE 5: NATURAL TRIGONOMETRIC FUNCTIONS

| Angle | | | | | Angle | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Degree | Radian | Sine | Cosine | Tangent | Degree | Radian | Sine | Cosine | Tangent |
| 0° | .000 | 0.000 | 1.000 | 0.000 | | | | | |
| 1° | .017 | .018 | 1.000 | .018 | 46° | 0.803 | 0.719 | 0.695 | 1.036 |
| 2° | .035 | .035 | 0.999 | .035 | 47° | .820 | .731 | .682 | 1.072 |
| 3° | .052 | .052 | .999 | .052 | 48° | .838 | .743 | .669 | 1.111 |
| 4° | .070 | .070 | .998 | .070 | 49° | .855 | .755 | .656 | 1.150 |
| 5° | .087 | .087 | .996 | .088 | 50° | .873 | .766 | .643 | 1.192 |
| 6° | .105 | .105 | .995 | .105 | 51° | .890 | .777 | .629 | 1.235 |
| 7° | 1.22 | .122 | .993 | .123 | 52° | .908 | .788 | .616 | 1.280 |
| 8° | .140 | .139 | .990 | .141 | 53° | .925 | .799 | .602 | 1.327 |
| 9° | .157 | .156 | .988 | .158 | 54° | .942 | .809 | .588 | 1.376 |
| 10° | .175 | .174 | .985 | .176 | 55° | .960 | .819 | .574 | 1.428 |
| 11° | .192 | .191 | .982 | .194 | 56° | .977 | .829 | .559 | 1.483 |
| 12° | .209 | .208 | .978 | .213 | 57° | .995 | .839 | .545 | 1.540 |
| 13° | .227 | .225 | .974 | .231 | 58° | 1.012 | .848 | .530 | 1.600 |
| 14° | .244 | .242 | .970 | .249 | 59° | 1.030 | .857 | .515 | 1.664 |
| 15° | .262 | .259 | .966 | .268 | 60° | 1.047 | .866 | .500 | 1.732 |
| 16° | .279 | .276 | .961 | .287 | 61° | 1.065 | .875 | .485 | 1.804 |
| 17° | .297 | .292 | .956 | .306 | 62° | 1.082 | .883 | .470 | 1.881 |
| 18° | .314 | .309 | .951 | .325 | 63° | 1.100 | .891 | .454 | 1.963 |
| 19° | .332 | .326 | .946 | .344 | 64° | 1.117 | .899 | .438 | 2.050 |
| 20° | .349 | .342 | .940 | .364 | 65° | 1.134 | .906 | .423 | 2.145 |
| 21° | .367 | .358 | .934 | .384 | 66° | 1.152 | .914 | .407 | 2.246 |
| 22° | .384 | .375 | .927 | .404 | 67° | 1.169 | .921 | .391 | 2.356 |
| 23° | .401 | .391 | .921 | .425 | 68° | 1.187 | .927 | .375 | 2.475 |
| 24° | .419 | .407 | .914 | .445 | 69° | 1.204 | .934 | .358 | 2.605 |
| 25° | .436 | .423 | .906 | .466 | 70° | 1.222 | .940 | .342 | 2.747 |
| 26° | .454 | .438 | .899 | .488 | 71° | 1.239 | .946 | .326 | 2.904 |
| 27° | .471 | .454 | .891 | .510 | 72° | 1.257 | .951 | .309 | 3.078 |
| 28° | .489 | .470 | .883 | .532 | 73° | 1.274 | .956 | .292 | 3.271 |
| 29° | .506 | .485 | .875 | .554 | 74° | 1.292 | .961 | .276 | 3.487 |
| 30° | .524 | .500 | .866 | .577 | 75° | 1.309 | .966 | .259 | 3.732 |
| 31° | .541 | .515 | .837 | .601 | 76° | 1.326 | .970 | .242 | 4.011 |
| 32° | .559 | .530 | .848 | .625 | 77° | 1.344 | .974 | .225 | 4.331 |
| 33° | .576 | .545 | .839 | .649 | 78° | 1.361 | .978 | .208 | 4.705 |
| 34° | .593 | .559 | .829 | .675 | 79° | 1.379 | .982 | .191 | 5.145 |
| 35° | .611 | .574 | .819 | .700 | 80° | 1.396 | .985 | .174 | 5.671 |
| 36° | .628 | .588 | .809 | .727 | 81° | 1.414 | .988 | .156 | 6.314 |
| 37° | .646 | .602 | .799 | .754 | 82° | 1.431 | .990 | .139 | 7.115 |
| 38° | .663 | .616 | .788 | .781 | 83° | 1.449 | .993 | .122 | 8.144 |
| 39° | .681 | .629 | .777 | .810 | 84° | 1.466 | .995 | .105 | 9.514 |
| 40° | .698 | .643 | .766 | .839 | 85° | 1.484 | .996 | .087 | 11.43 |
| 41° | .716 | .658 | .755 | .869 | 86° | 1.501 | .998 | .070 | 14.30 |
| 42° | .733 | .669 | .743 | .900 | 87° | 1.518 | .999 | .052 | 19.08 |
| 43° | .751 | .682 | .731 | .933 | 88° | 1.536 | .999 | .035 | 28.64 |
| 44° | .768 | .695 | .719 | .966 | 89° | 1.553 | 1.000 | .018 | 57.28 |
| 45° | .785 | .707 | .707 | 1.000 | 90° | 1.571 | 1.000 | .000 | ∞ |

Frequently, the use of sin, cos, tan ratios plays an important role in solving right-triangle problems such as (a) finding unknown sides of right triangles and (b) finding unknown angles of right triangles. In both types of problems, the unknown quantity must be defined in terms of the defining equations for the trig ratios. Examples E and F illustrate how an unknown side and an unknown angle of a right triangle are found.

**EXAMPLE E: FINDING UNKNOWN SIDES OF A RIGHT TRIANGLE**

Given: A right triangle has the values shown in the adjoining sketch.




Find: The length of sides A and B.

Solution: Recall the definitions of sine and cosine:

$$\sin 42^\circ = \frac{\text{side A}}{113 \text{ ft}} \qquad \cos 42^\circ = \frac{\text{side B}}{113 \text{ ft}}$$

Use Table I or an electronic calculator:

$$\sin 42^\circ = 0.669 \qquad \cos 42^\circ = 0.743$$

Therefore —

$$0.669 = \frac{\text{side A}}{113 \text{ ft}} \qquad 0.743 = \frac{\text{side B}}{113 \text{ ft}}$$

$$0.669\ (113 \text{ ft}) = \text{side A} \qquad 0.743\ (113 \text{ ft}) = \text{side B}$$

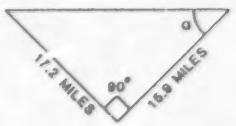
$$75.6 \text{ ft} = \text{side A} \qquad 84 \text{ ft} = \text{side B}$$

The solution can be checked independently by application of the Pythagorean theorem:

$$(75.6 \text{ ft})^2 + (84 \text{ ft})^2 = (113 \text{ ft})^2$$

**EXAMPLE F: FINDING AN UNKNOWN ANGLE OF A RIGHT TRIANGLE**

Given: A right triangle has values shown in adjoining sketch.




Find: The angle α in degrees.

Solution: The side opposite angle $\alpha$ is known (17.2 miles), and the side adjacent to $\alpha$ is known (15.9 miles). These two sides define the tangent ratio of angle $\alpha$; therefore —

$$\tan \alpha = \frac{17.2 \text{ miles}}{15.9 \text{ miles}}$$

$$\tan \alpha = 1.08$$

Use the tangent column of Table I to locate value near 1.08; then read angle:

$$\alpha = 47.2 \text{ degrees}$$

In Example F, the angle was found by use of a known tangent value. This can be accomplished approximately by use of Table I for sines, cosines, and tangents, or by use of an electronic calculator. Before this operation can be accomplished, the inverse trigonometric functions must be defined.

Often, as in Example F, the "inverse" operation is desirable; namely, given a trig ratio, one needs to find the associated angle. Inverse trigonometric ratio definitions frequently are written in the form shown here:

| INVERSE RATIO | | RATIO |
|---|---|---|
| $\sin^{-1}(\frac{A}{B}) = \theta$ | where | $\sin \theta = \frac{A}{B}$ |
| $\cos^{-1}(\frac{C}{B}) = \theta$ | where | $\cos \theta = \frac{C}{B}$ |
| $\tan^{-1}(\frac{A}{C}) = \theta$ | where | $\tan \theta = \frac{A}{C}$ |

The inverse ratio is indicated by a superscript (which is not a power) on the ratio name:

$\sin^{-1}(\frac{A}{B})$ is read as "the angle whose sine is $\frac{A}{B}$"

$\cos^{-1}(\frac{C}{B})$ is read as "the angle whose cosine is $\frac{C}{B}$"

$\tan^{-1}(\frac{A}{C})$ is read as "the angle whose tangent is $\frac{A}{C}$"

Example G contains several inverse trigonometric ratios.

**EXAMPLE G: INVERSE TRIGONOMETRIC RATIOS**

**Given:** (a) $\sin \alpha = 0.4226$.
(b) $\cos \beta = 0.7771$.
(c) $\tan \gamma = 0.3057$.
(d) $\tan \phi = 2.475$.

**Find:** (a) $\alpha$, (b) $\beta$, (c) $\gamma$, and (d) $\phi$.

**Solution:** (a) $\alpha = \sin^{-1}(0.4226) = 25°$
(b) $\beta = \cos^{-1}(0.7771) = 39°$
(c) $\gamma = \tan^{-1}(0.3057) = 17°$
(d) $\phi = \tan^{-1}(2.475) = 68°$

Many electronic calculators provide a means of obtaining inverse trigonometric ratios. Usually the ratio is entered into the calculator, and an [INV] button is pushed before pushing the respective trig ratio button [SIN], [COS], or [TAN]. The owner's manual for the calculator should give the proper sequence of operations. Example G can be used for practice with inverse trig ratios.

## VECTORS

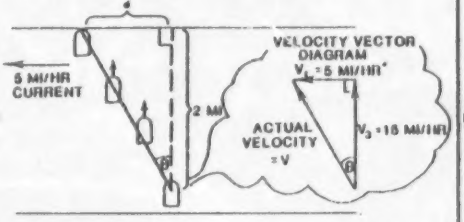
A vector is a quantity that must be specified by (1) a magnitude and (2) an angle relative to some chosen reference direction. Vectors represent many physical quantities, such as displacement, velocity, acceleration, force, and torque. Other physical quantities, such as work, energy, power, and resistance, are specified by an amount only and are referred to as "scalars."

When vector quantities are added, they yield another vector quantity that is not necessarily in the same direction as any of the original vectors. This situation is considered in Example H.

**EXAMPLE H: VECTOR ADDITION BY USE OF TRIG FUNCTIONS**

**Given:** A boat is moving at a speed of 15 mi/hr and aimed straight across a river flowing perpendicular to the boat's intended path, at a speed of 5 mi/hr. The river is 2 miles wide.

**Find:** (a) Distance from the starting point to the point at which the boat will land across the river.



(b) Velocity of the boat with respect to an observer on shore.

Solution: (a) Notice that the distance triangle and the velocity triangle are similar right triangles. The heading angle $\theta$ from the velocity diagram is –

$$\theta = \tan^{-1}\left(\frac{5 \text{ mi/hr}}{15 \text{ mi/hr}}\right)$$

$\theta$ is the same angle in the distance triangle. In the distance triangle –

$$\theta = \tan^{-1}\left(\frac{d}{2 \text{ mi}}\right)$$

$$\text{Therefore} - \left(\frac{5 \text{ mi/hr}}{15 \text{ mi/hr}}\right) = \tan\theta = \left(\frac{d}{2 \text{ mi}}\right)$$

$$\left(\frac{1}{3}\right) = \left(\frac{d}{2 \text{ mi}}\right)$$

$$d = 2/3 \text{ mile}$$

(b) Heading angle: $\theta = \tan^{-1}\left(\frac{5 \text{ mi/hr}}{15 \text{ mi/hr}}\right) = \tan^{-1}\left(\frac{1}{3}\right)$

$$\theta = 18.4 \text{ degrees}$$

The actual velocity of the boat can be found by use of the definitions of sin $\theta$, cos $\theta$, or Pythagorean theorem. Here the sin function is chosen:

$$\sin\theta = \frac{\text{velocity of river}}{\text{actual velocity}}$$

$$\sin\theta = \frac{v_r}{v_a}$$

since $\theta = 18.4$ degrees and $v_r = 5$ mi/hr

$$\text{Actual velocity} - v_a = \frac{5 \text{ mi/hr}}{\sin(18.4^\circ)}$$

$$= \frac{5 \text{ mi/hr}}{0.315}$$

$v_a = 15.84$ mi/hr in a direction 18.4 degrees downstream from straight across

A frequent application of trigonometric ratios occurs in the process of resolving a vector into two right-angle component vectors. Example H illustrates how two right-angle vectors add to give one resulting vector. (Remember the velocity vector diagram.) Resolving a vector into its right-angle components is the inverse problem. This inverse problem can be stated simply: Given a

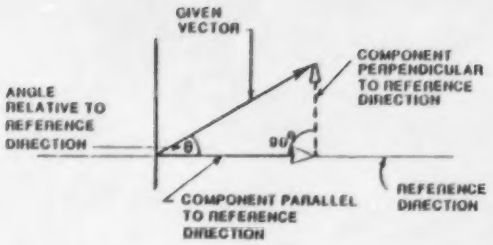



FIG. 11

vector and its angle relative to a reference direction, find two right-angle component vectors that would add "vectorially" to yield the given vector. One of these component vectors points in a direction parallel to the reference direction. These relationships are illustrated in Figure 11.

The right triangle formed by the vectors in Figure 11 is used as a reference to find the magnitudes of the component vectors. This vector triangle is shown symbolically in Figure 12. Application of the definition of sine θ, and cosine θ to the triangle in Figure 12 results in Equations 1 and 2.

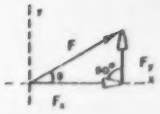



FIG. 12

$$\sin \theta = \frac{F_y}{F} \qquad \cos \theta \frac{F_x}{F}$$

$$\begin{bmatrix}\text{the magnitude}\\ \text{of component}\\ \text{opposite } \theta\end{bmatrix} = \begin{bmatrix}\text{magnitude}\\ \text{of vector}\end{bmatrix} \times \sin \theta$$

$$F_y = F \sin \theta \qquad (1)$$

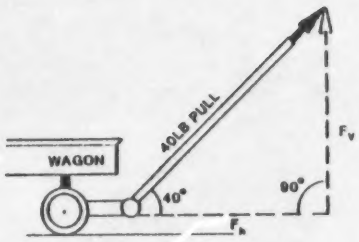
AND

$$\begin{bmatrix}\text{the magnitude}\\ \text{of component}\\ \text{adjacent to } \theta\end{bmatrix} = \begin{bmatrix}\text{magnitude}\\ \text{of vector}\end{bmatrix} \times \cos \theta$$

$$F_x = F \cos \theta \qquad (2)$$

Notice the relationship between the trigonometric ratio and the component desired:

| EXAMPLE I: FINDING VECTOR COMPONENTS BY USE OF TRIGONOMETRIC RATIOS | |
|---|---|
| Given: | A 40-lb pull at 40° is applied along the tongue of a wagon, as illustrated below. |




$F_v$ = amount of vertical lift caused by the 40-lb pull at 40°.

$F_h$ = amount of horizontal force tending to cause the wagon to roll.

Find: The vertical and horizontal components ($F_h$, $F_v$) of the 40-lb pull acting at 40 degrees to the horizontal direction.

Solution: Use Equations 1 and 2:

$F_v = F \sin \theta$ $\qquad$ $F_h = F \cos \theta$

$F_v = (40\ lb)(\sin 40°)$ $\qquad$ $F_h = (40\ lb)(\cos 40°)$

Obtain the trig ratio from Table 1 or from electronic calculator:

$F_v = (40\ lb)(0.6428)$ $\qquad$ $F_h = (40\ lb)(0.7660)$

$F_v = 26$ lb (rounded to nearest pound) $\qquad$ $F_h = 31$ lb (rounded to nearest pound)

These components are the combined affects on the wagon tongue caused by the single 40-lb force acting at 40 degrees. Notice that 26 lb and 31 lb do not add algebraically to give 40 lb. The components add vectorially to give a 40-lb force at 40°.

## SUMMARY

Trigonometry, which is concerned with the relations between the angles and sides of triangles and with functions based on these relations, is an indispensable mathematical tool in practically all branches of science and engineering. For example, many of the

problems from physics and mechanics involving force, velocities, and displacements are simplified by the application of trigonometry. Trigonometric measurement is indirect; for instance, with trigonometry one can measure the height of a cliff without ever climbing it and measure the width of a river without ever crossing it. Even in business and economics, in fact in any situation where the available data seem to suggest periodic phenomena, trigonometry can be a useful aid. Vectors are an extremely important tool in technology.

## ANGLES AND TRIANGLES

### MATERIALS

Protractor (0-180 x 1 degree).
Metric scale, flat plastic (1 ft length, 0.1 cm).
Electronic calculator with trig functions (optional).

### PROCEDURE

1. Using a protractor, measure θ, β, and γ; record below.

   θ = _____ degrees
   β = _____ degrees
   γ = _____ degrees

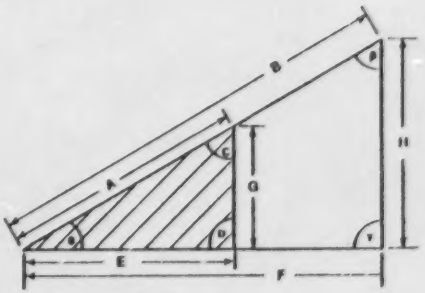



FIG. 13

2. Use a protractor to measure angle C and D; record below.

   C = _____ degrees
   D = _____ degrees

3. Use a metric scale to measure the following lengths labeled in Figure 13, and record them:

   A = _____ cm  B = _____ cm
   G = _____ cm  H = _____ cm
   E = _____ cm  F = _____ cm

4. Calculate the following ratios, using the measurements in Step 3:

   a. $\sin \theta = \frac{G}{A} = \frac{(\quad\quad)}{(\quad\quad)} =$ _____

   b. $\sin \theta = \frac{H}{B} = \frac{(\quad\quad)}{(\quad\quad)} =$ _____

   c. $\cos \theta = \frac{E}{A} = \frac{(\quad\quad)}{(\quad\quad)} =$ _____

d. $\cos \theta = \frac{F}{B} = \frac{(\quad\quad)}{(\quad\quad)} =$ _____

e. $\tan \theta = \frac{G}{E} = \frac{(\quad\quad)}{(\quad\quad)} =$ _____

f. $\tan \theta = \frac{H}{F} = \frac{(\quad\quad)}{(\quad\quad)} =$ _____

5. Use Table I or an electronic calculator to find the angle to the nearest degree for each of the trig ratios in Step 4.

a. $\theta =$ _____ degrees

b. $\theta =$ _____ degrees

c. $\theta =$ _____ degrees

d. $\theta =$ _____ degrees

e. $\theta =$ _____ degrees

f. $\theta =$ _____ degrees

6. Compare your answers in Step 5 to the measured angle $\theta$ in Step 1. How do they compare?

## SELF CHECK

1. Identify the following terms from the adjoining sketches:

a. An angle: B (example).

b. Right angle: _____.

c. Straight angle: _____.

d. Acute angle: _____.

e. Obtuse angle: _____.

f. A triangle: _____.

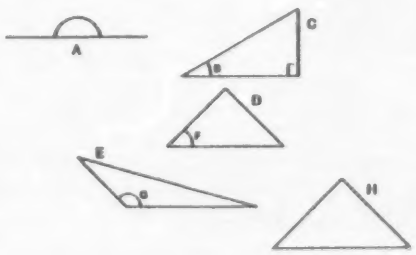
g. Right triangle: _____.

h. Acute triangle: _____.

i. Obtuse triangle: _____.

j. Similar triangles: _____

and _____.




2. Use a protractor to measure the following angles. Record degree of each under corresponding sketch.

3. Define the following terms; and use diagrams to illustrate definitions if needed:

   a. A degree.
   b. The sine of an angle.
   c. The cosine of an angle.
   d. The tangent of an angle.

4. From the data gathered in the Laboratory exercise, verify these facts for similar triangles:

   a. The ratio of corresponding sides of similar triangles are equal.

   $\frac{B}{A} =$ _____, $\frac{H}{G} =$ _____, $\frac{F}{E} =$ _____.

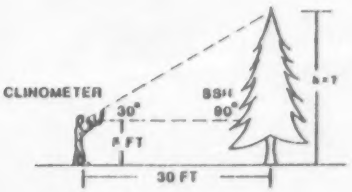
   b. All angles of a triangle add to equal 180°.

   $\angle\theta + \angle C + \angle D = ?$

   $\angle\theta + \angle B + \angle r = ?$

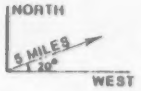
5. For the right triangle below, calculate the hypotenuse with the Pythagorean theorem:

6. Bob Crachett wants to know how tall the Christmas tree is before he chops it down to take home. He measures the angle with a clinometer as shown in the diagram, to be 30° while he stands 30 ft from the tree. From the ground to Bob's eye level is 5 ft. Calculate the height (h) of the tree for Bob.



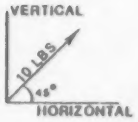

7. Resolve the following vectors into components along the indicated directions:

a.



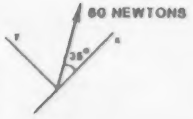

(along north and west)

b.




(along vertical and horizontal)

c.




(along x and y)

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Palmer, Claude Irwin and Bibb, Samuel F. Practical Mathemtics. 5th ed. New York: McGraw-Hill Book Company, 1970.

Paul, Richard S. and Shaevel, Leonard M. Essentials of Technical Mathematics. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1974

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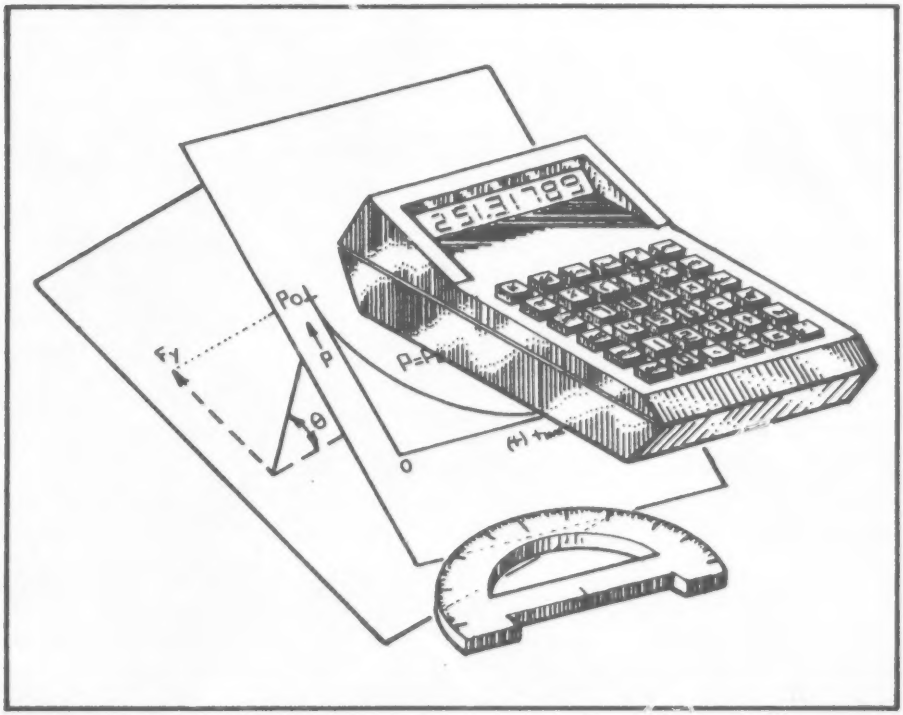
UNIFIED TECHNICAL CONCEPTS

MODULE S 5

VECTORS AND SCALARS




SUPPLEMENTARY

Technical Education Research Center-Southwest TERC-SW

# INTRODUCTION

Basic physical quantities such as speed, temperature, mass, and pressure require only a magnitude to specify their values. They are called "scalar quantities," or simply "scalars." Other basic physical quantities such as force, velocity, acceleration, and torque require both a magnitude and a direction to specify their values. They are called "vector quantities," or simply "vectors."

Scalars and vectors play important roles in science and engineering. To know the difference between them and to be able to describe them correctly when using them is essential. This module defines scalars and vectors and distinguishes carefully between them. Different methods of combining (adding or subtracting) scalars and vectors are outlined in detail. In the laboratory, the student will draw and resolve vectors.

# PREREQUISITES

Student should have completed one year of high school algebra.

# OBJECTIVES

Upon completion of this module, the student should be able to:

- List at least four common examples of the following:

  Scalar quantities.
  Vector quantities.

- Given a vector and a scalar quantity, graphically show the results when the vector is multiplied or divided by the scalar.

- Given two or more vector quantities, add and subtract the vectors graphically, with the aid of a scale and protractor, to obtain a resultant vector.

- Given a vector quantity with magnitude and direction specified, resolve the vector graphically, with the aid of a scale and protractor, into two components at right angles to each other.

"Scalar quantities" are defined as "quantities completely specified only by their magnitude (amount)." The number and its unit are all that is needed to represent a scalar quantity. Some examples of scalar quantities are mass (12 g), area (5 sq mi), volume (144 cu in), density (2.7 grams per cubic cm), distance (12 mi), speed (30 mi/hr). Any quantity represented as a count of objects, such as a dozen eggs in a box, money in a pocket, and a number of birds in a flock, is a scalar quantity. The student may have encountered such problems as those illustrated in Figure 1 while studying arithmetic in Elementary School. These problems are simple examples of scalar addition, in which the magnitudes (sizes) of the quantities are added together. In scalar subtraction, the magnitudes are subtracted from each other.

30 CENTS + 40 CENTS = 70 CENTS
40 APPLES - 30 APPLES = 10 APPLES

FIG. 1

In engineering and technical work, a different type of quantity often is used in problem solving. This quantity, called a "vector," has its own rules for addition and subtraction.

## VECTORS

A "vector quantity" is one that completely is specified by a magnitude and a direction. It consists, therefore, of a number, of a unit, and of a direction. Some examples of vector quantities are displacement (20 mi, North), velocity (180 mi/hr, 30° South of West), and force (100 lb, 30° above horizontal). Other examples of vector quantities are acceleration, momentum, torque, and electric fields. Vectors contain scalar magnitudes in their specifications. Several vector quantities often are confused with similar scalar quantities; however, the names of the quantities should distinguish betwen them. Two examples of scalar and vector terms often confused are shown in Table 1.

TABLE 1, SCALAR AND VECTOR QUANTITIES

| NAME OF SCALAR QUANTITY | NAME OF VECTOR QUANTITY |
|---|---|
| DISTANCE (30 MILES) | DISPLACEMENT (30 MILES, WEST) |
| SPEED OR RATE (60 MI/HR) | VELOCITY (60 MI/HR, 20° WEST OF NORTH) |

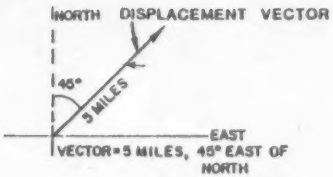



FIG. 2

The direction of the vector is an important aspect and always should be included when specifying the vector. The direction of a vector usually is given relative to a reference direction, such as an X-axis or an eastward line. Figure 2 consists of a vector diagram that represents a displacement. The angle 45° specifies the vector's direction, or heading, in the right-angle coordinate system of NORTH, EAST, WEST, SOUTH. Another coordinate system frequently used is the common X-Y axes at 90° to one another.

In a scale drawing of a vector quantity, the length of the line represents the magnitude of the vector quantity. The orientation of the line on the page represents its "line of direction." The arrow-tip drawn at one end of the vector indicates the "sense" of the vector along the line of direction. These characteristics are illustrated in Figure 3.

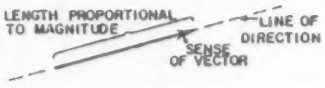



FIG. 3

Sometimes the arrow-tip is omitted by mistake from the end of the vector. When this error occurs, confusion arises concerning which way the vector points along the line of direction (Figure 4).

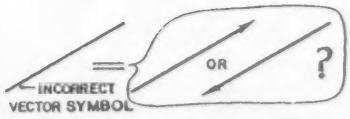



FIG. 4

In engineering and technical work, vector quantities often are added by a process called "vector addition." When vector addition is performed, both the magnitude and the direction of the quantities must be considered. In Figure 5, the vector sum is —

$$\overrightarrow{40 \text{ miles}} + \overrightarrow{30 \text{ miles}} = \overrightarrow{50 \text{ miles}}$$

The arrow is placed above the quantity to indicate that it is a vector quantity. It is also possible to combine or add a 40-mi and a 30-mi displacement to obtain a different resultant. Other possible combinations are —

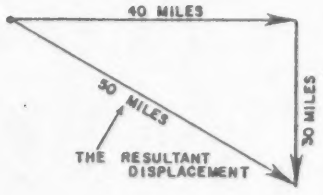



FIG. 6

vector sum: $\overrightarrow{30 \text{ miles}} + \overrightarrow{40 \text{ miles}} = \overrightarrow{70 \text{ miles}}$,

as shown in Figure 6,

and vector sum: $\overrightarrow{40 \text{ miles}} - \overrightarrow{30 \text{ miles}} = \overrightarrow{10 \text{ miles}}$,

as shown in Figure 7. A fourth possibility is shown in Figure 8.

vector sum: 40 miles + 30 miles = 40 miles.

FIG. 6

When a 40-mi displacement and a 30-mi displacement are added, the magnitude of the resultant may vary from a minimum of 10 mi to a maximum of 70 mi, as demonstrated in Figures 6 and 7. The direction must be taken into account when displacement vectors (or any set of vectors) are combined by vector addition. Vectors can be added together to find a resultant vector — a single vector that produces the same effect as the original vectors added together.

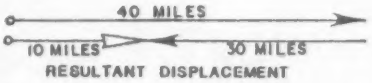



FIG. 7

The subtraction of vectors is simple once the addition of vectors is understood. Consider Figure 8, where the vector addition of 40 miles + 30 miles = 40 miles was illustrated. Suppose that they were subtracted according to 40 miles - 30 miles = Resultant. The subtraction is handled as an addition as follows: 40 mies - 30 miles = 40 miles + (-30 miles) = Resultant. Since the magnitude and direction of the vector is known to be 30 miles, the vector (-30 miles) is just the reverse. Thus, the correct subtraction process would appear as shown in the upper half of Figure 9. When 40 mi + 30 mi are added as shown in Figure 8 or the lower half of Figure 9, the resultant is 40 mi in the direction down and to the right, as shown. When the same two vectors are subtracted, the resultant is about 60 mi up and to the right, as shown.

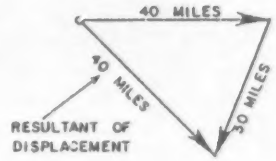



FIG. 8

Addition of a vector and a scalar or subtraction of one from the other would be meaningless; however, vector quantities can be multiplied or divided by scalars in a particular way. The process of multiplying a vector quantity by a scalar quantity yields a resulting vector in the same direction with a different magnitude than the original vector. This process is illustrated in Figure 10. Notice that the new magnitude is merely the original magnitude multiplied by the scalar.

FIG. 9

In order to draw on paper a vector that represents a

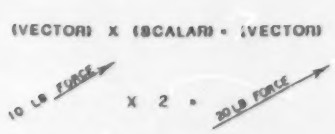



FIG. 10

physical quantity expressed in some other units (miles, pounds of force, miles per hour), one must choose a scale with which to represent the vector as a line of some convenient length on the page. The scale establishes a simple ratio of length units to the units of the vector quantity, as illustrated in Figure 11.

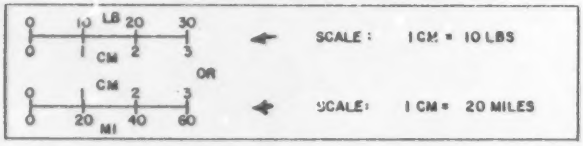



FIG. 11

A vector quantity can be converted to a line vector by simple multiplication of the vector by the appropriate scale ratio. This process is shown in Example A.

**EXAMPLE A: CONVERSION OF A VECTOR QUANTITY TO A GRAPHIC VECTOR**

Given: A vector has a displacement of 100 mi EAST and a scale of 1 cm = 20 mi.

Find: Length of the graphic vector in cm.

Solution: Multiply the magnitude of the vector by the scale ratio:

$$(100 \text{ mi}) \left(\frac{1 \text{ cm}}{20 \text{ mi}}\right) = 5 \text{ cm}$$

Therefore, in a graphical vector problem using the given scale, a 100-mi vector point EAST would be drawn on the page as a graphic vector 5 cm in length, pointing EAST.

## ADDING VECTORS BY THE GRAPIC (POLYGON) METHOD

In adding vectors by the graphical method, the magnitude and direction of each vector is indicated by an arrow-tipped line drawn to some specified scale. In using the graphical polygon method, a ruler is employed to measure lengths, and a protractor

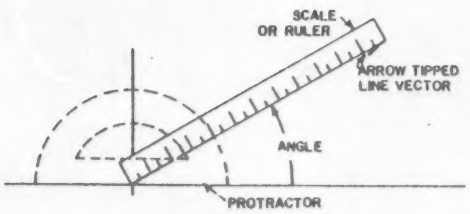



FIG. 12

to measure angles, as shown in Figure 12. The step-by-step procedure for adding vectors by the polygon method is as follows:

1. Choose a suitable scale, and determine the length of the arrow which will represent the vector according to the specified scale.

2. Choose an origin or starting point on the paper, and draw the arrow representing the first vector. This arrow must be of proper length and must point in the appropriate direction indicated by its angle. The "tail" of the arrow always is positioned at the origin, or starting point.

3. The arrow of the second vector is drawn so that its tail is joined to the head of the first vector. The arrow of the second vector also must be drawn to scale and must point in the correct direction indicated by its angle.

4. If there is a third vector to be added, its tail is joined to the head of the second vector and correctly oriented. This process of joining tail to head is continued until the magnitude and direction of all vectors have been represented.

5. The arrow representing the resultant vector is drawn with its tail at the origin (starting point) and its head joined to the head of the last vector.

6. The resultant arrow indicates the magnitude and direction of the resultant. Measurement of the resultant arrow with the ruler and the protractor gives the magnitude and direction of the resultant vector.

7. The length of the resultant vector is converted back to the units of the original vectors by multiplying by the appropriate scale factor.

If only two vectors are added by this method, as in Figure 8,

the resultant vector forms the third side of a triangle (a three-sided polygon). The polygon method can be extended to find the resultant of the addition of any number of vectors. Example B illustrates the application of this step-by-step procedure to a vector addition problem.

**EXAMPLE B: ADDING VECTORS BY GRAPHICAL METHOD**

Given: A ship travels 100 mi due North on the first day of a voyage. It then travels 80 mi northeast on the second day and 130 mi due East on the third day.

Find: The resultant displacement after three days.

Solution:

1. Establish a suitable scale of 20 mi = 1 cm.

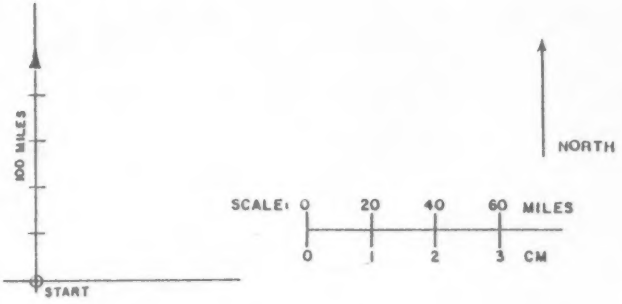
2. Convert all vector magnitudes in the problem to graphical line lengths:

$$100 \text{ mi} \times \left(\frac{1 \text{ cm}}{20 \text{ mi}}\right) = 5 \text{ cm (North)}$$

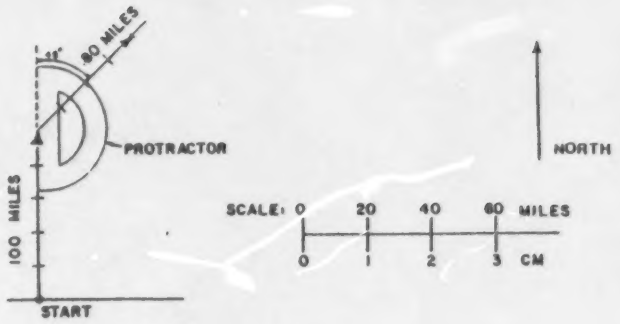
$$80 \text{ mi} \times \left(\frac{1 \text{ cm}}{20 \text{ mi}}\right) = 4 \text{ cm (Northeast)}$$

$$130 \text{ mi} \times \left(\frac{1 \text{ cm}}{20 \text{ mi}}\right) = 6.5 \text{ cm (East)}$$

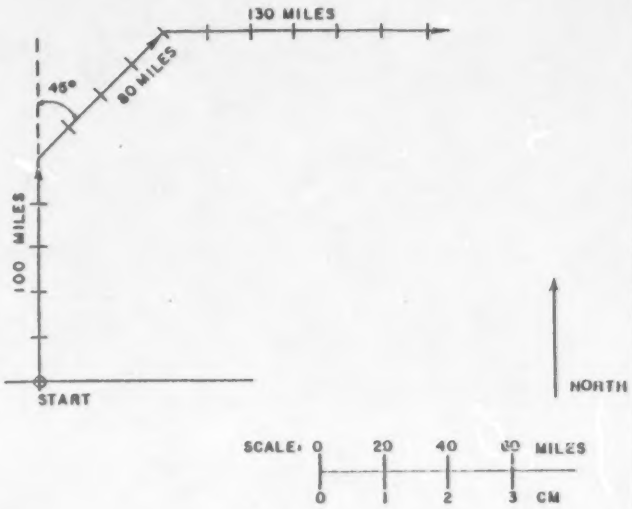
3. Choose a starting point, and draw the first 5-cm (100 mi) vector in its direction (North), as indicated by the arrow:



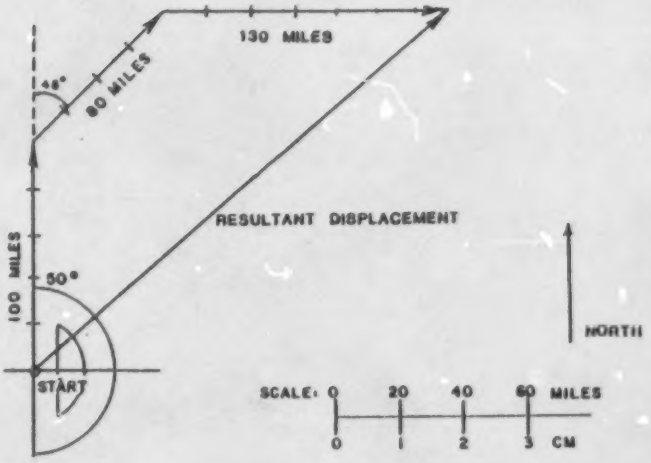

4. Measure the angle northeast (45° in direction East of North), and draw the second 80-mi displacement (4 cm) as shown:




5. Draw the 130-mi vector (6.5 cm) in a due east direction by attaching its tail to the head of the 80-mi vector as shown:



6. Draw the resultant vector from the start (origin) to the tip of the 130-mi (6.5 cm) vector:




7. Measured with a ruler, the resultant vector is 12.3 cm in length; therefore, the magnitude of the resultant displacement is —

$$12.3 \text{ cm} \times \left(\frac{20 \text{ mi}}{1 \text{ cm}}\right) = 246 \text{ mi.}$$

The direction is found with a protractor, and the angle is measured relative to North and found to be 50°. The resultant displacement, therefore is —

246 mi in a direction 50° East of North.

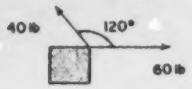
Steps 1, 2, 3, 4, and 5 of this solution illustrated the building of the graphical solution. The graphical solution actually would be done on one diagram and appear as in Step 6. Notice that on any graphical solution, the scale used must be shown near the vector polygon diagram.

The graphical polygon method can be used to find the resultant of all kinds of vectors. In Example C, the method is used to find the resultant of two force vectors.

## EXAMPLE C: GRAPHICAL ADDITION OF FORCE VECTORS

**Given:** A 60-lb pull and a 40-lb pull are applied to an object. The angle between these two forces is 120°.

**Find:** The resultant force.

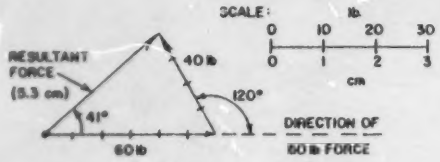



**Solution:** 1. Establish a scale 1 cm = 10 lb:

$$60 \text{ lb} \times \left(\frac{1 \text{ cm}}{10 \text{ lb}}\right) = 6 \text{ cm}$$

$$40 \text{ lb} \times \left(\frac{1 \text{ cm}}{10 \text{ lb}}\right) = 4 \text{ cm}$$

2. Addition of these two vectors by the graphical method produces the following diagram:



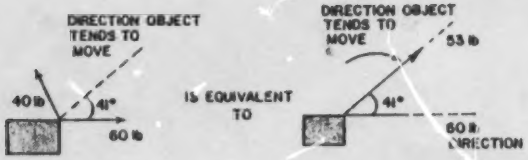

3. The resultant force vector = 5.3 cm; therefore,

$$5.3 \text{ cm} \times \left(\frac{10 \text{ lb}}{1 \text{ cm}}\right) = 53 \text{ lb.}$$

4. Using a protractor, the angle that the resultant vector makes with the direction of 60-lb force is 41°.

5. The resultant force is 53 lb at an angle of 41° with the 60-lb force direction.

6. This means that the net effect of the 53-lb force at the angle of 41° would be the same on the object as the combined affect of the two given forces:




## GRAPHICAL COMPONENTS

Often, the replacement of one vector with an equivalent set of two vectors is advantageous; this process is the reverse of the situation shown in Step 6 of Example B. The two replacement vectors, called "components," are desired most often at right angles to each other. Two such components of a vector are shown in Figure 13.

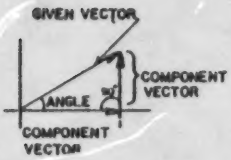
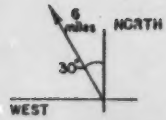



FIG. 13

The two components add vectorially to produce the given vector. The directions of the components usually are chosen to be parallel to the coordinate system directions, such as NORTH, EAST, SOUTH, WEST, and HORIZONTAL-VERTICAL. The given vector then is said to be "resolved" into the two desired components.

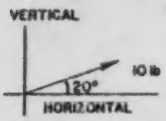
To find graphically the magnitude of the components of a given vector, the vector first must be scaled as in the polygon method and oriented in the proper direction relative to one of the chosen coordinate system directions. Examples of vectors whose components might be desired are shown in Figures 14 a, b, and c.



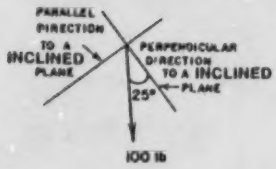

(A)

FIG. 14

Second, a line must be drawn from the tip of the given vector perpendicular to either of the two coordinate axes. The sides of the right triangle produced can be measured with a ruler. The lengths of the two sides, converted according to the proper scale, represents the two components. Example D illustrates the technique.



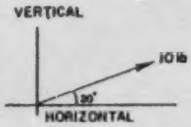

(B)



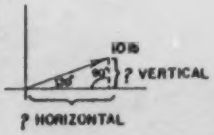

(C)

**EXAMPLE D: GRAPHICAL RESOLUTION OF A GIVEN VECTOR INTO ITS PERPENDICULAR COMPONENTS**

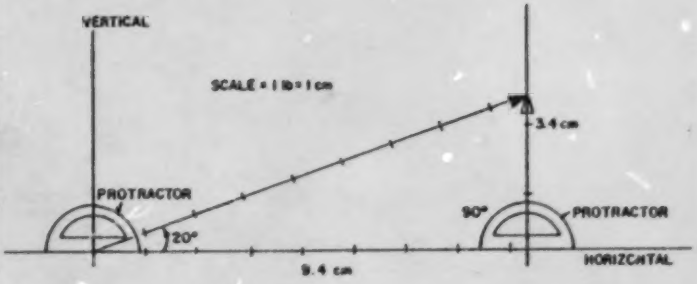
Given: A 10-lb force vector acts 20° above the horizontal to the right.




Find: The magnitude of its components along the horizontal and vertical directions.




Solution: 1. Scale the 10-lb force using 1 cm = 1 lb on a right-angle coordinate system drawn on the page at 20° as shown below:




2. Use a protractor to construct a line from the tip of the scaled vector to meet the horizontal direction at 90°.

3. Measure the sides of the right triangle formed:

   HORIZONTAL COMPONENT: 9.4 cm
   VERTICAL COMPONENT: 3.4 cm

4. Convert to original units by the scale factor:

$$9.4 \text{ cm} \times \left(\frac{1 \text{ lb}}{1 \text{ cm}}\right) = 9.4 \text{ lb}$$

$$3.4 \text{ cm} \times \left(\frac{1 \text{ lb}}{1 \text{ cm}}\right) = 3.4 \text{ lb}$$

5. The components have these magnitudes:

   HORIZONTAL: 9.4 lb
   VERTICAL: 3.4 lb

In Example D, a 9.4-lb force acting horizontally on an object and a 3.4-lb force acting vertically on the same object have the same effect as a 10-lb force acting on the object at 20° above the horizontal. Often, a physical meaning can be given to these components. For example, if the 10 lb force of Example D were applied to a child's wagon as illustrated in Figure 15, the horizontal component of 9.4 lb may be considered to be the propelling effect of the 10-lb force that tends to make the wagon roll. The vertical component of 3.4 lb may be considered to be the lifting affect of the 10-lb force applied at the base of the tongue of the wagon. This lift would tend to

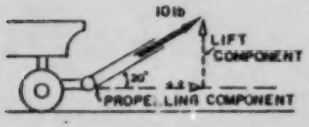



FIG. 15

make the front wheels lift up as the wagon was pulled.

## SUMMARY

A scalar quantity can be specified only by magnitude (size); whereas, a vector quantity is specified by both magnitude and direction. Some examples of quantities often represented as vectors are velocity, force, and acceleration. Examples of scalar quantities are time, temperature, and speed.

Vectors are represented by arrows, the length of which represents magnitude of the vector quantity. The arrowhead indicates the direction of the vector quantity. Vectors can be added to produce the "resultant"; such added vectors are called "component vectors."

Vectors and scalars find wide application in technology; for example, in physics, electronics, navigation, etc.

## DRAWING AND RESOLVING VECTORS

### MATERIALS

Several sharp pencils.
Notebook paper or graph paper (5 sqs to the inch).
A thin, flat plastic ruler (1 ft length, English-Metric Scale with metric scale to 1 mm).
A see-thru plastic protractor (0-180 x 1°) similar to STAEDTLER 568 50-10.

### PROCEDURE

Solve the following problems by the graphical methods illustrated in this module. Draw a sketch of the physical situation, and illustrate the vectors involved in each problem. Sketch simple arrow-tipped lines.

1. A 100-lb push toward the north and a 200-lb push toward the east act upon an object. Determine the magnitude and direction of the resultant force by the graphical method.

2. The following three forces are acting upon an object:

   a. 1000 lb, horizontally to the right.
   b. 1500 lb, downward and to the left at an angle of 30° below the horizontal.
   c. 500 lb, upward and to the right at an angle of 53° above the horizontal.

   Find the resultant by the polygon method.

3. A ship that can travel 30 mi/hr in still water is headed due west. A 10 mi/hr current is flowing northeast. How fast and in what direction will the ship travel?

4. a. Resolve a 200-mi, 30° east of north displacement vector into its easterly and northerly components.
   b. Resolve a 100-mi, due west displacement vector into its northerly and westerly components.
   c. Resolve a 80-mi, 60° west of south displacement vector into its southerly and westerly components.
   d. Add the three displacement vectors in parts a, b, and c above by the graphical polygon method, and determine

the magnitude and direction of the resultant vector.

e. Find the components of the resultant vector along the North-South line and along the East-West line.

f. Show the scalar addition of the North-South components of a, b, and c that would result in the East-West component of the resultant found in e. How do they compare?

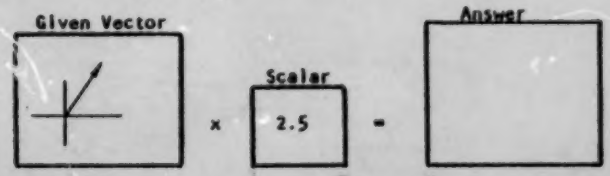
5. Discuss how vectors could be applied to several physical quantities in technology.

## SELF CHECK

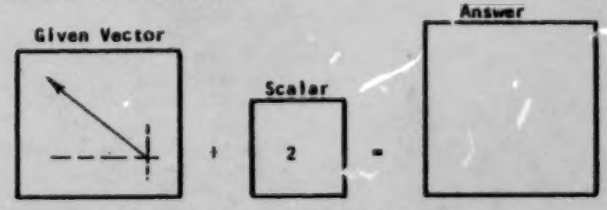
1. Define a scalar quantity, and list at least four examples of scalar quantities.

2. Define a vector quantity, and list at least four examples of vector quantities.

3. In the space provided below, show the result of multiplying the given vector by the scalar quantity 2.5. Use a protractor and a scale.



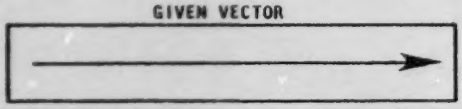

4. In the space provided below, graphically draw the result of dividing the given vector by the scalar quantity 2. Use a protractor and a scalar.




5. The vector drawn below represents a force vector drawn to a scale of 40 lbs = 1 cm. Measure the length of the vector with

a metric scale, and state the magnitude of the force vector in pounds.

GIVEN VECTOR



MAGNITUDE = _____ LBS

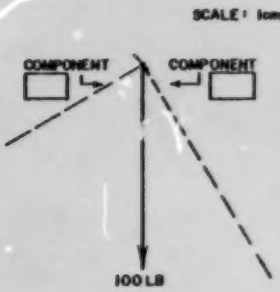
6. On a piece of graph paper, using protractor and scale, work the following problem:

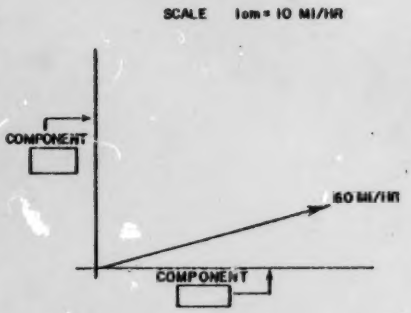
   Explorers leave their base and travel the following distances on three successive days:

   1st day: 60 miles — 50° east of north.
   2nd day: 40 miles — 90° east of north (due east).
   3rd day: 50 miles — 120° east of north.

   How far and in what direction must the explorers travel in order to return to their base on the fourth day?

7. In each of the vectors illustrated below, sketch graphically their components, and find the magnitudes of each component.



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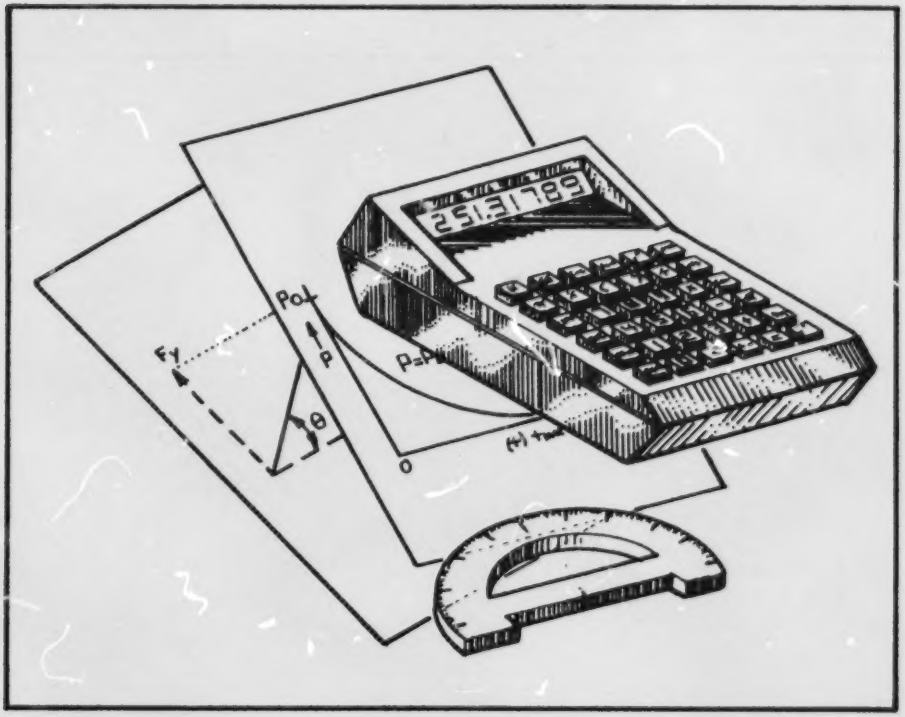
UTC

UNIFIED TECHNICAL CONCEPTS

MODULE S 6

DIMENSIONAL ANALYSIS




SUPPLEMENTARY

Technical Education Research Center-Southwest TERC-SW

## INTRODUCTION

The method of handling units in calculations is called "dimensional analysis." It should be used in studying and solving problems in physics and engineering. Attention paid to units will reinforce the physics principles and will help the problem-solving process.

In this module, some advantages of dimensional analysis are presented, along with some of the techniques for handling units in calculations. In the laboratory, the student will calculate and convert with dimensional analysis.

## PREREQUISITES

Student should have completed one year of high school algebra.

## OBJECTIVES

Upon completion of this module, the student should be able to:

- Distinguish between fundamental and derived units.
- Given the steps in the solution of an equation, evaluate the correctness of the units in the solution.
- Given an equation and the units of each term, determine the units of the solution.
- Given the necessary equivalents, convert a physical quantity from one set of units to another using the ratio method.

## SYSTEMS OF UNITS

Two systems of units commonly are used today in science and technology in this country. The systems are (1) the familiar English system and (2) the International Systems (SI) of units, which incorporates units of the older Metric system. Each of these systems contains fundamental, or basic, units that are defined in such a way as to be independent of all other units, and derived units that may be expressed and defined in terms of the fundamental units of the system.

### FUNDAMENTAL UNITS

The fundamental units of the English system are the foot (length), the second (time), the pound (force), and the degree Fahrenheit (temperature). These fundamental units and the units derived from them are used to express all physical quantities in the English system. There are no English units for electrical energy systems.

The fundamental units in the SI system are the meter (length), the second (time), the kilogram (mass), the ampere (electric current), the kelvin (temperature), and the candela (light intensity). This set of basic units and the units derived from them are employed to express all physical quantities in the SI system.

### DERIVED UNITS

Units that are a comparison of two or more quantities and that require a multi-word label are referred to as "derived units." Miles per hour (mi/h) and pounds per square inch ($lb/in^2$) are examples of derived units. Many derived units are important enough to have special names, but they still are defined in terms of basic units.

In the SI system, the unit of force is called a "newton" (N). One newton of force is the force necessary to produce an acceleration of one meter per second squared ($m/s^2$) when applied to a mass of one kilogram (kg). The following expression shows the definition of this derived unit in terms of basic units:

$$1\ N = 1\ \frac{kg \cdot m}{s^2}$$

In the English system, the unit of force (pound) is a basic unit; and the unit of mass is derived. One slug is the amount of mass that will experience an acceleration of one foot per second squared ($ft/s^2$) when acted upon by a one-pound (lb) force. The

following equation is the definition of a slug in terms of basic units:

$$1 \text{ slug} = \frac{\text{lb}}{\text{ft/s}^2} = \frac{\text{lb s}^2}{\text{ft}}$$

Other derived units with special names are the newton-meter ("joule"), the volt/amp ("ohm"), and the newton/meter$^2$ ("pascal"). All derived units may be written as algebraic combinations of basic units, as illustrated in Example A.

**EXAMPLE A: DERIVED AND FUNDAMENTAL UNITS**

Given: (a) 1 watt (W) = 1 joule/second (J/s).
(b) 1 pascal (Pa) = 1 newton/meter$^2$ (N/m$^2$).

Find: Each of the above units in terms of basic SI units.

Solution: (a) $1 \text{ W} = 1 \text{ J/s}$

$= 1 \frac{\text{N·m}}{\text{s}}$ since $1 \text{ J} = 1 \text{ N/m}$

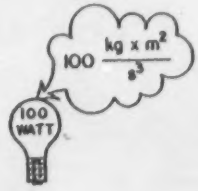
$= 1 \left(\frac{\text{kg·m}}{\text{s}^2}\right)\left(\frac{\text{m}}{\text{s}}\right)$ since $1 \text{ N} = 1 \frac{\text{kg·m}}{\text{s}}$

$1 \text{ W} = 1 \frac{\text{kg·m}^2}{\text{s}^3}$

(b) $1 \text{ Pa} = 1 \text{ N/m}^2$

$= 1 \left(\frac{\text{kg·m}}{\text{s}^2}\right)\left(\frac{1}{\text{m}^2}\right)$

$1 \text{ Pa} = 1 \frac{\text{kg}}{\text{m·s}^2}$




Notice that in Example A the unit abbreviations are treated as algebraic variables. The processes of canceling and combining are accomplished according to the rules of algebra.

## DIMENSIONLESS QUANTITIES

Dimensionless quantities often are encountered in calculations. Dimensionless quantities are treated simply as pure numbers without units. They usually are the result of a comparison, in the form of a ratio, of quantities having the same units. Because units are treated as algebraic quantities, any unit divided by itself simply cancels out.

A good example of this is a supplementary unit of angular measure in the SI system called a "radian" (rad). Figure 1 shows an angle of 1 rad (about 57.3°). An angle measured in radian is defined by Equation 1.

$$\theta = \frac{s}{r}$$

FIG. 1

where

θ = angle in radians.
s = length of arc subtended on a circle.
r = radius of circle.

Clearly, since s and r are both in units of length, a radian is simply a ratio and carries no "real" units. The term "radian" is attached to the ratio to indicate that it expresses the size of an angle. "Radian" is dropped in all calculations because it is only a ratio.

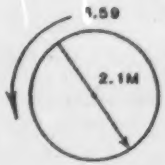
Some other dimensionless quantities are given in Table 1.

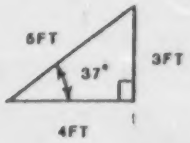
TABLE 1. DIMENSIONLESS QUANTITIES

$$\pi = \frac{\text{circumference of circle}}{\text{diameter of circle}} = \frac{6.59\ \text{m}}{2.1\ \text{m}} = 3.14$$

$$\text{Tan } 37° = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{3\ \text{ft}}{4\ \text{ft}} = 0.7536$$

$$\text{Specific gravity of iron} = \frac{\text{density of iron}}{\text{density of water}} = \frac{7900\ \frac{\text{kg}}{\text{m}^3}}{1000\ \frac{\text{kg}}{\text{m}^3}} = 7.9$$



## CHECKING AN EQUATION DIMENSIONALLY

Operations performed with units are similar to the operations performed with numbers: therefore, if the units of an answer are not appropriate, some error has occured in the problem-solving process. One way to spot an erroneous equation is to check the dimensions of all of its terms. In any legitimate physical equation, dimensions of all terms on one side of an equation must be equal to dimensions of all terms on the other side of the equation.

Consider the equation $s = vt + (1/2)at^2$ where s represents distance, v represents speed (distance/time), t represents time, and a represents acceleration (distance/time$^2$). Substitution of metric units into the equation gives the following:

$$s = \left(\frac{\text{meters}}{\text{sec}}\right)(\text{sec}) + \left(\frac{\text{meters}}{\text{sec}^2}\right)(\text{sec}^2)$$

Like units cancel, resulting in these units:

$$s = \text{meters} + \text{meters} = \text{meters}$$

The equation is now termed a "dimensionally consistent" equation because s should have the unit "meters." A correct equation always is dimensionally consistent.

Sometimes equations are dimensionally correct, but the terms may be expressed in units incapable of addition or subtraction. For example, "1 meter + 40 centimeters" is dimensionally correct because it is a length added to a length; however, a conversion to one or the other of the units is necessary before the terms can be added [i.e., 1 meter + 0.4 meter = (1 + 0.4) meter = 1.4 meter].

## CONVERSION BETWEEN UNITS

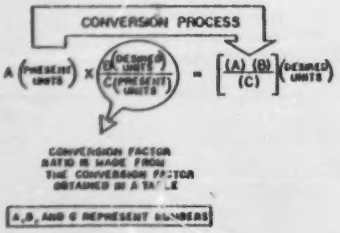

CONVERSION PROCESS
A (PRESENT UNITS) X (DESIRED UNITS / PRESENT UNITS) = [(A)(B)/(C)] (DESIRED UNITS)
CONVERSION FACTOR RATIO IS MADE FROM THE CONVERSION FACTOR OBTAINED IN A TABLE
A, B, AND C REPRESENT NUMBERS

FIG. 2

Dimensional analysis also is useful in solving unit-conversion problems. Figure 2 illustrates a one-step conversion process.

Examples of the conversion process are shown in Example B.

Notice that in Example B, arrangement of the conversion factor ratio is dictated by the units. In converting 6 ft to $\frac{?}{(12\text{ in} = 1\text{ ft})}$ in, suppose the conversion factor had been arranged incorrectly as follows:

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$$6 \text{ ft} = (6 \text{ ft})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) = \frac{(6)(1)}{(12)} = 0.5 \text{ ft}^2/\text{in}$$

The units of $ft^2/in$ are certainly not the desired units.

**EXAMPLE B: ONE-STEP UNIT CONVERSIONS**

Given: Units to be converted:

(a) Convert 6 ft to ? in
(1 ft = 12).

(b) Convert 12 in to ? cm
(1 in = 2.54 cm).

(c) Convert 21 ℓ ? gal (ℓ = liter)
(1 gal = 3.785 ℓ).

Find: Converted units.

Solution: (Use conversion process in Figure 1.)

(a) $(6 \text{ ft})\left(\frac{12 \text{ in}}{1 \text{ ft}}\right) = \left[\frac{(6)(12)}{(1)}\right] \text{ in} = 72 \text{ in}$

(b) $(12 \text{ in})\left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right) = \left[\frac{12\ (2.54)}{1}\right] \text{ cm} = 30.5 \text{ cm}$

(c) $(21\ \ell)\left(\frac{1 \text{ gal}}{3.785\ \ell}\right) = 5.548 \text{ gal}$

Use of a conversion ratio to change units makes possible the discovery of errors when they are made.

Conversion processes can be conducted in a step-by-step manner or in a "string" manner. The string method is particularly useful when converting derived quantities. An example of the string method is the problem of converting 30 miles/hour to ? ft/s. Start by writing the original measurement:

$$30 \frac{\text{mi}}{\text{h}}$$

Multiply by the conversion factor ratio necessary to convert miles to feet (5280 ft = 1 mile):

$$30 \frac{\text{mi}}{\text{h}} = \left(30 \frac{\text{mi}}{\text{h}}\right)\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)$$

(arranged this way in order that the mile units will cancel)

Without multiplying the numbers, multiply this string by the conversion ratios necessary to convert hours to minutes (1 hour =

60 minutes) and minutes to seconds (1 minute = 60 seconds):

$$30\ \frac{mi}{h} = (30\ \frac{mi}{h})(\frac{5280\ ft}{1\ mi})(\frac{1\ h}{60\ min})(\frac{1\ min}{60\ s})$$

(arranged this way in order that the hour and minute units will cancel)

Cancellation of the units reveals that the only units left are ft/s, which are the desired units. At this time, the numerical calculation is made:

$$30\ \frac{mi}{h} = \frac{(30)(5280)(1)(1)}{(1)(1)(60)(60)}\ \frac{ft}{s} = 44\ \frac{ft}{s}$$

Therefore: 30 miles hour = 44 feet/second.

The same conversion factor can be used repeatedly as many times as indicated by the units. Example C shows 140 cubic inches being converted to cubic centimeters.

**EXAMPLE C: CONVERTING UNITS**

Given: A volume (V) of 140 $in^3$.

Find: This volume expressed in $cm^3$.

Solution:
$$140\ in^3 = (140\ in^3)(\frac{2.54\ cm}{1\ in})(\frac{2.54\ cm}{1\ in})(\frac{2.54\ cm}{1\ in})$$
$$V = 140\ (2.54)^3\ cm^3$$
$$V = 2294.2\ cm^3$$

## SUMMARY

The units on both sides of an equation must be equivalent. Dimensional analysis is a method of solving problems of and locating errors based upon this fact. When dimensional analysis is used to check the solution of a problem, the procedure is to determine if the units on each side of the equation are equivalent and appropriate. If they are, the solution may be correct. (Dimensional analysis will not reveal math errors.) If they are not, the solution is incorrect. A solution for distance in m/s, for example, cannot be correct.

Dimensional analysis also can be used as a key to the methods of solving problems; for example, since the units of velocity are distance units divided by time units, velocity is calculated by dividing distance by time.

Careful attention to the units involved in a problem can provide useful problem-solving information and can reduce errors.

## CALCULATION AND CONVERSION WITH DIMENSIONAL ANALYSIS

### MATERIALS

Pencil.
Paper.

### PROCEDURES

1. List below the names and symbols of units of length, time, mass, force, and velocity in both English and SI units. Identify which are basic units, and give the derived units in terms of basic units.

| | English | SI |
|---|---|---|
| length | _____ | _____ |
| time | _____ | _____ |
| mass | _____ | _____ |
| force | _____ | _____ |
| velocity | _____ | _____ |

2. Given the equation $T = 2\pi\sqrt{\frac{I}{K}}$ where I has units: $kg \cdot m^3$ and K has units: $kg \cdot m/s^2$, what are the units of T? _____ _____.

3. In the study of fluids, an equation for Reynolds number ($N_R$) is given by $N_R = \rho v d/\eta$, where $\rho$ has units of $kg/m^3$, v has units of m/s, d has units of m, and $\eta$ has units of $kg/m \cdot s$, what are the units of $N_R$? _____.

4. Intensity of a sound wave is given by the equation, $I = p^2/2\rho_o v$, where p has units of $N/m^2$, $\rho_o$ has units of $kg/m^3$ and v has units of m/s, what are the units of I? _____ _____.

5. A newton (N) = $kg \cdot m/s^2$, and a joule = $kg \cdot m^2/s^2$. What is the unit result of dividing a joule by a newton? _____ _____.

6. One volt = joule/coulomb and one ampere = C/s. What is the unit result of multiplying one volt by one ampere? (Hint: a J/s is equal to a watt.) _____.

7. Calculate $v_f$, given this equation:

$$v_f^2 = v_o^2 + 2gs$$

where

$v_o = 20$ cm/s.
$s = 5$ cm.
$g = 980$ cm/s$^2$.

8. In the following equations, substitute appropriate units for the symbols. Check for dimensional consistency, identifying the incorrect equations. (The symbol s is in meters; $v_f$, $v_o$, and v are in m/s; a is in m/s$^2$; t is in s.)

a. $s = \frac{v}{t}$

b. $a = s/t$

c. $v_f^2 = v_o - 2as$

d. $s = v_o + 1/2\ at^2$

9. Use the Conversion Factors Table below to complete the following statements:

a. 35 knots = _____ furlongs/fortnight

b. 45 miles per hour = _____ cm/shake

c. 210 km/s = _____ ft/min

d. 11 cm = _____ furlongs

e. 8 mi$^2$ = _____ ft$^2$

Conversion Factor Table

1 mile = 5280 ft = 8 furlongs
1 meter = 3.281 ft
1 meter = $10^{10}$ angstroms (A)
1 meter = $10^6$ micron (μ)
1 fermi = $10^{-15}$ m
1 cm = 2.54 in
12 in = 1 ft
1 h = 60 min
1 min = 60 s
1 day = 24 h
1 fortnight = 14 days
1 knot = 1.151 mi/h
1 shake = $10^{-8}$ s

1. List the fundamental units of the English system of units and the SI system of units.

2. Write the equivalent expressions for the following units in terms of basic units:
   a. Slug.
   b. Newton.
   c. Joule.
   d. Watt.
   e. Newton-meter.

3. Given the equation $F = \frac{1}{2} kx^2$, where k has units of $kg/m \cdot s^2$ and x has units of m, determine the units of F.

4. Calculate mass density ($\rho_m$) of a right circular cylinder using the equation —

$$\rho_m = \frac{m}{\pi r^2 h}$$

where
$\rho_m$ = mass density.
m = mass = 360 g.
r = radius = 3 cm.
h = height = 10 cm.

5. 1 volt = 1 J/C, 1 A = C/s, and 1 Ω = 1 V/A. Determine the equivalent of 1 Ω expressed in units of J, C, and s.

6. Using the table of conversion factors provided, work the following multiple-choice problems. Circle the correct answer.

| | | |
|---|---|---|
| 1 Btu = 252 cal | 1 At/m = $1.257 \times 10^{-2}$ gauss | 365 days = 1 yr |
| 1 lb = 453.6 | 1 esu = $2.655 \times 10^{-9}$ At/m | 24 h = 1 day |
| 1 C° = 1.8 F° | 1 bar = $10^5$ pascals | 60 min = 1 h |
| 1 C° = 1 K° | 1 psi = 6895 pascals | (kg = kilogram) |
| 1 cal = 4.184 J | 1 century = 100 yr | |

   a. The number of $\frac{\text{Btu}}{\text{lb/F°}}$ equivalent to 1.987 $\frac{\text{cal}}{\text{g/C°}}$ is —
   (1) 1.987. (2) 408,800. (3) 6.438. (4) 3.576.

   b. The number of $\frac{\text{joule}}{\text{kg/K°}}$ equivalent to 1.987 $\frac{\text{cal}}{\text{g/C°}}$ is —
   (1) 8.314. (2) 8314. (3) 4619. (4) 1000.

   c. The number of minutes in 1.0 microcentury is —
   (1) 0.0365. (2) 0.876. (3) 53. (4) 3180.

   d. A certain magnetic field has an intensity of $4.5 \times 10^5$ esu. This intensity expressed in gauss is—
   (1) $8.2 \times 10^{-16}$. (2) $15 \times 10^{-8}$. (3) $1.5 \times 10^{-2}$. (4) 15.

e. The production of synthetic diamonds from graphite requires a pressure of about 150 kilobars. This pressure expressed in psi is —
(1) $2.2 \times 10^6$. (2) $1.5 \times 10^5$. (3) 22. (4) $1 \times 10^{14}$.

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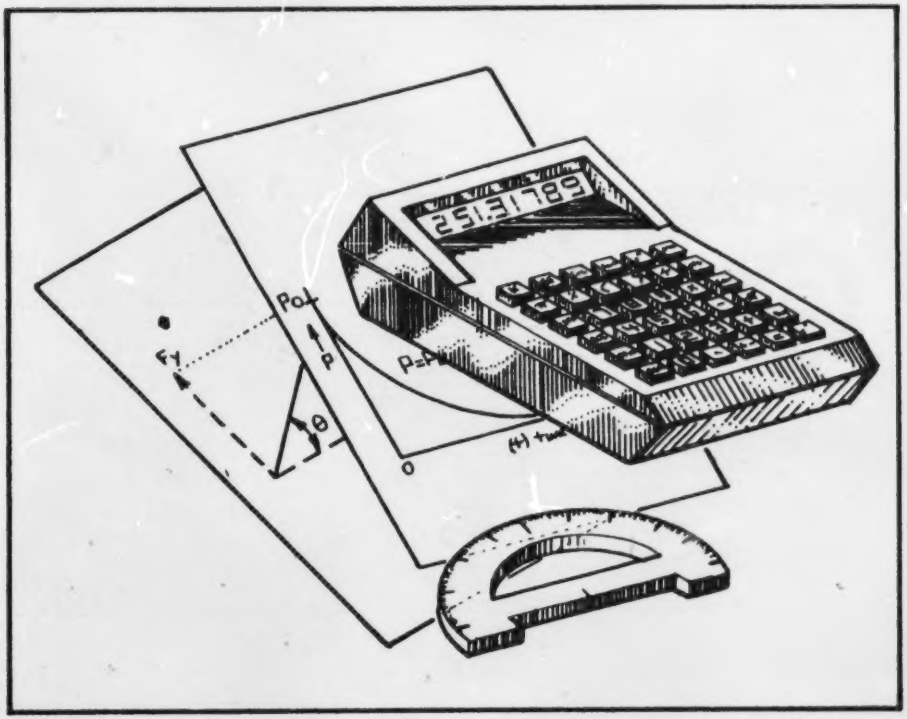
UTC

UNIFIED TECHNICAL CONCEPTS

MODULE S 7

# READING AND DRAWING GRAPHS




SUPPLEMENTARY

Technical Education Research Center-Southwest

TERC-SW

## INTRODUCTION

Technical and scientific work often is a study of the relationship between two interacting physical quantities. If one quantity is varied, the other quantity is affected. A graphical display of the values of two physical quantities shows clearly how one quantity is related to another. Incorrect data values are detected more easily when observed in graphical form than when seen as a table of values. A graph of experimental data serves frequently as a tool and as a communication device in the professional work of engineers and technicians. For the solution of many technological problems, the graphical method is sufficiently accurate, faster than algebraic methods, and less susceptible to error.

In this module, the student will be presented a proper and generally-acceptable method of preparing laboratory data in graphical form. In the laboratory, the student will examine the criteria of acceptability for graphs.

## PREREQUISITES

Student should have completed one year of high school algebra.

## OBJECTIVES

Upon completion of this module, the student should be able to:

- Draw a technically-acceptable graph from a table of experimental data suitable for plotting on rectangular-coordinate graph paper. The resulting graph may be linear or nonlinear.

## BASIC STEPS FOR GRAPHING EXPERIMENTAL DATA

1. Examine the tabular data, and identify which variable is the independent variable and which is the dependent variable. In an experiment, the values of the independent variable are set, or controlled, by the technician or experimenter. The dependent variable is measured as a result of setting a particular value of the independent variable. In graphing practice, the independent variable customarily is plotted along the abscissa (horizontal axis), and the dependent variable along the ordinate (vertical axis), as shown in Figure 1.

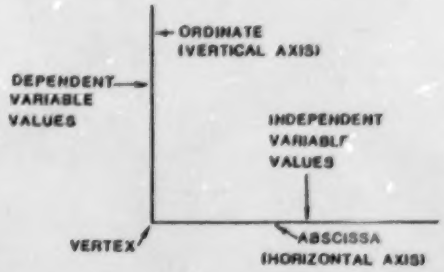



FIG. 1

Sometimes, by examination of the incremental steps in the values of the variables, the independent and dependent variables can be identified. The independent variable often will be incremented in even steps, and the independent variable values will appear to have a random incrementation, as shown in Figure 2.

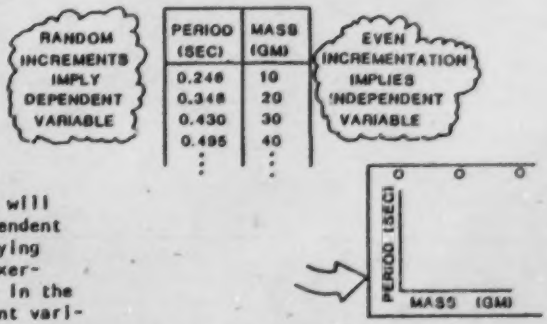



FIG. 2

In this module, the exercises will aid in the identification of independent and dependent variables by identifying them in a customary way. In any exercise, the variable mentioned first in the instruction problem is the dependent variable; and the second variable mentioned is the independent one. An example is, "plot the variation of voltage with current." Here, voltage is the dependent variable and current is the independent variable.

2. Select standard-quality, commercial graph paper as listed in the materials section of this module.

3. Select the position of the paper for which the rectangular coordinate axes will be drawn. Acceptable positions are illustrated in Figure 3. The holes of the paper, if any, should be on the left hand side of the paper or at the top of the page.

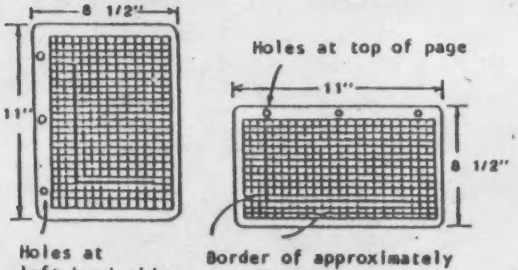

8 1/2"
11"
Holes at top of page
11"
8 1/2"
Holes at left hand side of page
Border of approximately one or two major grid divisions (For titles, lettering, and scaling)

FIG. 3

The axes are drawn as shown in Figure 3, leaving a space of one or two major grid divisions from the border. This space will be used for title and scale lettering.

4. For each variable select a scale that conveys the best information to the best advantage. The choice of scales is the most important and, at the same time, the most difficult part of graph construction. Scale selection does more to shape the picture of the variable relationship than all other steps; it makes the difference between an accurate picture and a misleading one. For this reason, the discussion of scales must be somewhat lengthy; however, an attempt is made to prevent many common student errors before they occur. Some helpful hints are as follows:

a. Make a trail computation to select the scale length that will "fit" on the page and will have divisions easy to plot. Choose a length that will spread the data as much as possible along the axis on which the variable is to be plotted, while at the same time arranging that the units per division are 1, 2, 5, 10, or multiples of these numbers, and sometimes 4. Never use 3, 7, or 9, since these values would cause a graph to be difficult to plot and to read.

b. The divisions on the vertical axis need not be the same as those on the horizontal axis. Each scale should be chosen independently as described in a.

c. Construct the axes and scales in such a manner that all quadrants required to display the positive and negative ranges of the data are available.

d. The lettering of intermediate scale values over the scale range requires some care for clarity. Scale values always increase away from the lower left-hand corner of a first-quadrant graph; that is, upward or to the right. Intermediate scale values should be chosen for ease of rapid location of data points and should not be crowded together. Some common errors are shown in Figure 4.

e. Print the numerals for the scale, using the coordinate lines of the graph paper as lettering guides. The numerals should be 2 or 3 divisions high, depending upon the coordinate paper. Center the numerals about the scale coordinates that they identify.

f. Zero is the most desirable beginning of the ordinate and abscissa axes unless this would compress the curve unnecessarily. The origin is placed in the lower left-hand corner except for cases in which both positive and negative

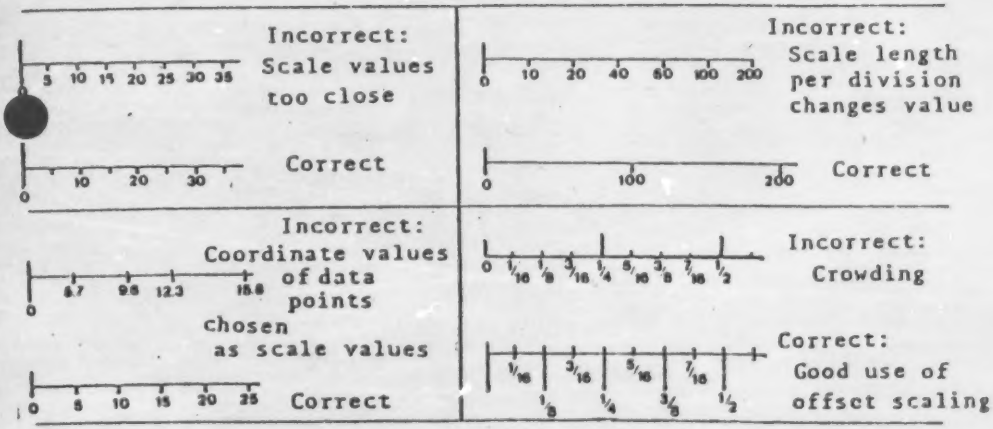



FIG. 4

values are to be plotted. In such cases, the origin should be located in order that all desired values can be shown. Frequently, the intersection of the two axes in the left-hand corner of the graph is labeled with "even, chosen" values slightly less than the lowest values in the ranges of the two variables, as shown in Figure 5.

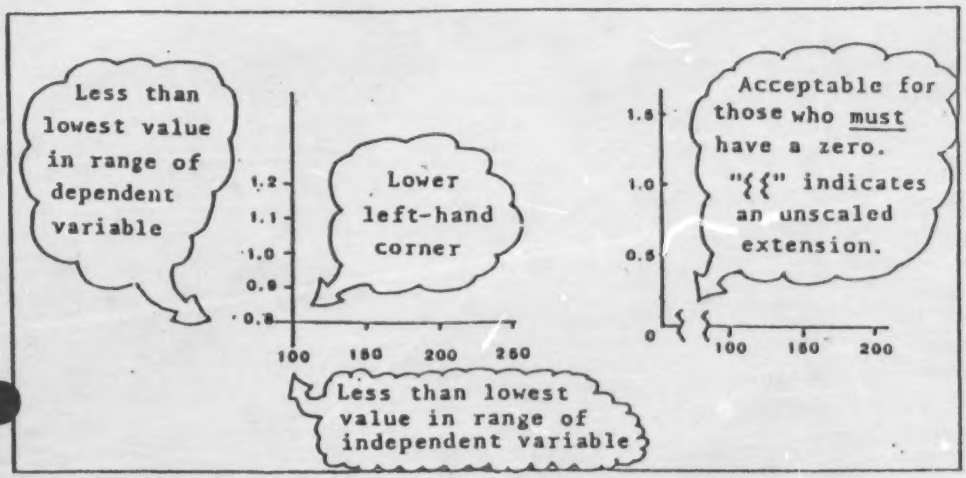

Less than lowest value in range of dependent variable
Lower left-hand corner
Acceptable for those who must have a zero.
"{{" indicates an unscaled extension.
Less than lowest value in range of independent variable

FIG. 5

g. In the space outside the scale, print a clear, complete label. Letter each scale on the graph so that the scale label can be read from the lower left-hand corner. A good practice is to write all labels in capital letters, although some authorities capitalize only the first word and proper names. In either case, a consistent style should be chosen. The scale label includes the name of the quantity plotted and its unit, separated by a comma or a parenthesis. Abbreviate all units in standard form. Figure 6 shows examples of scale labeling.

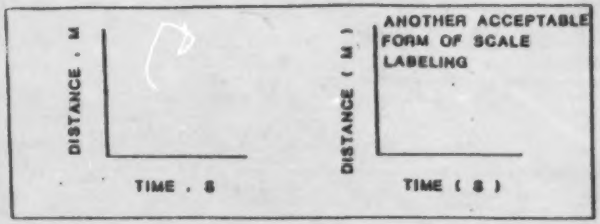



FIG. 6

5. LOCATION OF DATA POINTS. Locate each data point in its its proper place with a fine, tiny pencil dot, and draw a circle not more than 1/16 inch in diameter around the point. When multiple curves are plotted on one sheet, the points for each curve may be identified by other symbols, such as squares, diamonds, or other suitable geometric figures.

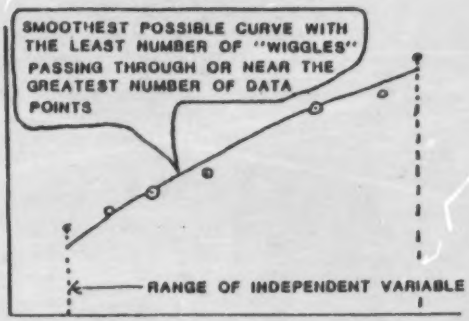

SMOOTHEST POSSIBLE CURVE WITH THE LEAST NUMBER OF "WIGGLES" PASSING THROUGH OR NEAR THE GREATEST NUMBER OF DATA POINTS
RANGE OF INDEPENDENT VARIABLE

FIG. 7

6. DRAWING THE CURVE. When plotted, experimentally-determined data often show a scattering of points about some smooth curve drawn through the points, as shown in Figure 7. In experimental work, points usually do not fall exactly upon a smooth curve, because of errors and inaccuracies that exist in the measurement process. A light, pencilled, free-hand line will aid in locating the average; but the final line should be drawn smoothly and clearly with a flexible curve or other drawing aid. In drawing the final curve, do not draw the line through the symbols that enclosed the plotted points. Stop at the perimeter of each symbol (i.e., —⊙—⊙—⊙—⊙—⊙). Do not extend the curve beyond the the range of the plotted data points.

7. TITLE OF GRAPH. The title of the graph should include the names of plotted quantities and should include other descriptive information, such as the value of relevant quantities held fixed during the experiment, names of equipment, date the data was obtained, where the data was obtained, and any other descriptive information about the graph. (See Figure 8.)

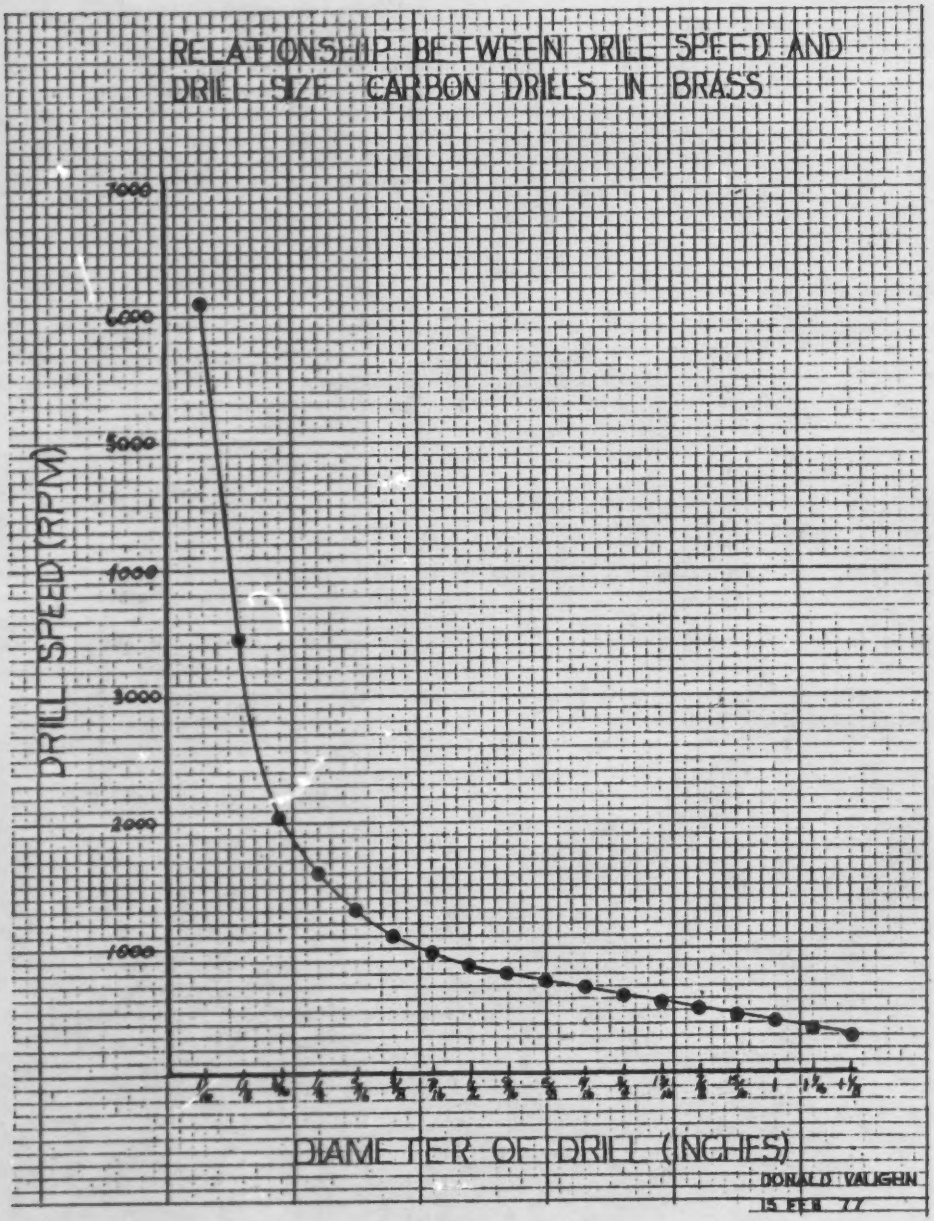

RELATIONSHIP BETWEEN DRILL SPEED AND
DRILL SIZE CARBON DRILLS IN BRASS
DRILL SPEED (RPM)
7000
6000
5000
4000
3000
2000
1000
DIAMETER OF DRILL (INCHES)
DONALD VAUGHN
15 FEB 77

FIG. 8

The title should be placed in such a position on the sheet that it will not interfere with the curve. The title section of a display graph usually is placed across the top of the sheet. Simple graphs that comprise parts of reports frequently have the title in either the lower right quadrant or the upper left quadrant.

8. Finally, the name of the person preparing the graph and the date that the graph is plotted should be placed in the lower right-hand corner of the sheet.

## SUMMARY

Graphs demonstrate how one quantity is related to another. The visual display furnished by graphs simplifies many relationships that might be less obvious when stated in equation form. The graphical solution generally is sufficiently accurate and, in addition, is less susceptable to error than algebraic methods.

The technician will have opportunity to use graphs in (1) plotting data and in (2) interpretating the graphical data presented by others in reports, manuals, textbooks, articles, etc. (Many UTC Modules employ graphs in the Discussion, Laboratory, and Self Check.)

## CRITERIA OF ACCEPTABILITY FOR GRAPHS

### MATERIALS

One package of rectangular coordinate graph paper, 10 x 10 to the inch or 10 millimeters to the centimeter. (8 1/2 x 11.)
One flat plastic ruler (1 foot long).
One No. 2 pencil with eraser.
One black ballpoint or fine black felt-tip pen.
One flexible curve drawing aid or a set of French curves.

### PROCEDURE

From the Self Check section of this module, choose any two exercises, and correctly present the data in graphical form according to the following criteria of acceptability. Use this set of criteria as a checklist for better organization of graphing activities.

1. Graph will be produced on one side of graph paper only.

2. Graph should be a full page presentation, with graph occupying approximately 3/4 of page area.

3. Final copy is prepared totally in ink. Location of data points and drawing of curve can be sketched in preliminarily with light pencil.

4. The independent variable is scale on abscissa; dependent variable is scaled on ordinate.

5. The scaling of the axis should be —

   a. Chosen to encompass range of the variable.
   b. Chosen in least-count increments of 1, 2, 5, or 10 square widths per unit for ease of data plotting.
   c. Numbered neatly, without crowding, and in the same, constant increments (for linear scale).
   d. Chosen in such a manner that the length of scale is neither unduly compressed (too short), possibly masking the behavior of the data nor unduly elongated (too

lengthy), possibly leaving too little space for all data points to be plotted.

e. Arranged in such a manner that at least two significant figures can be read from the scale.

6. Coordinate axes are labeled with variable names (not symbols), followed by a comma and by the unit (may be abbreviated). Use grid lines as lettering guides, and use capital letters (2 or 3 squares high) for 10 x 10 to inch grid.

7. Data points are located by small circles drawn around the data points. The diameter of these circles should be large enough to be seen easily without exceeding 0.4 cm.

8. The smoothest possible curve is drawn through the vicinity of the data points in order that as many points as possible fall on or near the curve. Curves should not be drawn through the data-point symbols.

9. The title of the graph should include the names of the plotted quantities and should include other suitable information that would help describe the graph.

10. The title should be placed on the sheet, but should not interfere with the curve. The title section of graphs frequently is placed across the top or bottom of the graphing sheet.

11. The general overall appearance should be neat, orderly, and easy to read and to interpret.

12. Why is a graph usually interpreted more easily than an equation?

## SELF CHECK

1. Plot a graph showing the relationship of normal barometric pressure of air to altitude. Plot values up to and including 15,000 ft. (Suggestion: Plot the independent variable along the longer length of graph paper.)

| Altitude (Feet above sea level) | Normal Barometric Pressure (Inches of Mercury) |
|---|---|
| 0 | 29.95 |
| 500 | 29.39 |
| 1000 | 28.86 |
| 1500 | 28.34 |
| 2000 | 27.82 |
| 2500 | 27.32 |
| 3000 | 26.82 |
| 4000 | 25.84 |
| 5000 | 24.9 |
| 6000 | 24.0 |
| 7000 | 23.1 |
| 8000 | 22.2 |
| 9000 | 21.4 |
| 10000 | 20.6 |
| 15000 | 16.9 |

2. The following data was taken in the laboratory for a 75-watt, gas-filled, tungsten-filament light bulb. Plot a resistance-voltage curve.

| Voltage (volts) | Resistance (ohms) |
|---|---|
| 10 | 47.5 |
| 20 | 77.5 |
| 30 | 100.3 |
| 40 | 119.0 |
| 50 | 132.6 |
| 60 | 144.2 |
| 70 | 160.2 |
| 80 | 170.0 |
| 90 | 178.3 |
| 100 | 189.0 |
| 110 | 200.1 |

3. Plot a graph showing the variation of temperature with electric current through a heating coil. The following data were taken in the laboratory:

| Current (amp) | Temperature (°Celsius) |
|---|---|
| 0.0 | 0.0 |
| 0.46 | 0.5 |
| 1.05 | 1.2 |
| 1.50 | 2.0 |
| 2.06 | 5.1 |
| 2.20 | 7.7 |
| 2.35 | 8.8 |

4. For a fairly clean iron pipe, 1 inch in diameter, and water at a temperature of about 70°F, the following losses of head pressure in inches of water per 1000 feet of pipe were calculated for water velocities ranging from 6 inches per second to 30 inches per second.

| Velocity of water in pipe (inches per second) | Loss of head pressure (inches of water per 1000 feet) |
|---|---|
| 6 | 20 |
| 12 | 80 |
| 18 | 180 |
| 24 | 300 |
| 30 | 440 |

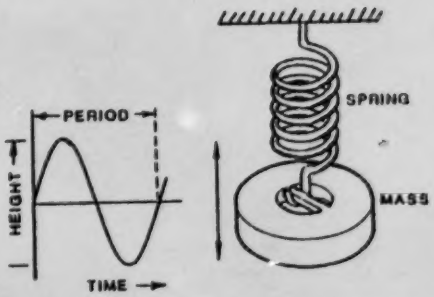
Plot a graph of the loss of head pressure versus water velocity for this pipe.

5. Plot the relation between magnetic flux density (B) in Kilolines per square centimeter and magnetizing force (H) in gilberts per centimeter for a specimen of tool steel. The graph will form what customarily is called a "B-H curve."

| Magnetic flux density (B) (Kilolines per $cm^2$) | Magnetizing force (H) (Gilberts per centimeter) |
|---|---|
| 9.00 | 27.1 |
| 11.80 | 54.2 |
| 13.02 | 81.3 |
| 13.75 | 108.4 |
| 14.09 | 135.5 |
| 14.22 | 162.6 |
| 14.66 | 189.7 |
| 14.86 | 216.8 |
| 14.98 | 243.9 |
| 15.23 | 271.0 |
| 15.35 | 298.1 |
| 15.57 | 325.2 |

6. Plot a graph for the following experimental data, showing the relation between the Period in seconds and the Mass in grams of a vibrating spiral spring.

| Period (Seconds) | Mass (grams) |
|---|---|
| 0.246 | 10 |
| 0.348 | 20 |
| 0.430 | 30 |
| 0.495 | 40 |
| 0.570 | 50 |
| 0.650 | 70 |
| 0.740 | 90 |
| 0.810 | 110 |
| 0.900 | 130 |
| 0.950 | 150 |




A PERIOD IS THE TIME FOR ONE CYCLE OF VIBRATING MOTION

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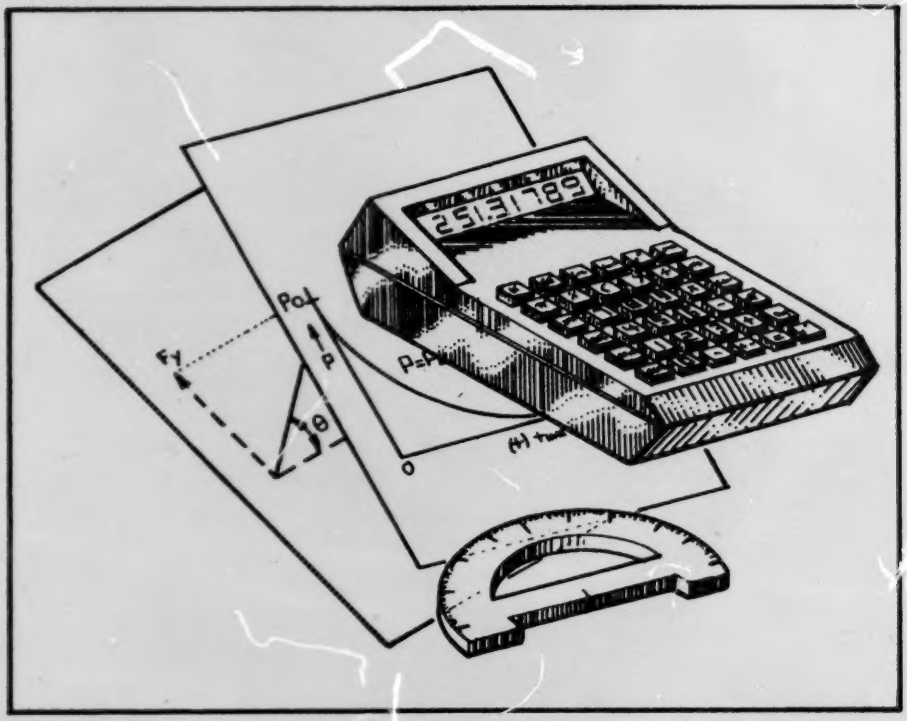
UNIFIED TECHNICAL CONCEPTS

MODULE S 8

# PRECISION, ACCURACY, AND MEASUREMENT




SUPPLEMENTARY

Technical Education Research Center-Southwest

TERC-SW

## INTRODUCTION

The ability to make precise and accurate measurements is essential to science and technology. Modern life would be impossible without standards of measurement and the knowledge and equipment to apply them. This module discusses the process of measurement. Particular attention is devoted to the accuracy with which measurements can be made and the methods of analyzing and reporting this accuracy. In the laboratory, the student will perform length measurements and will calculate the limits of error for direct and indirect measurements.

## PREREQUISITES

Student should have completed one year of high school algebra.

## OBJECTIVES

Upon completion of this module the student should be able to:

- Distinguish between the following pairs:

  Counting and measurement.
  Precision and accuracy.
  Schematic and random errors.
  Direct and indirect measurements.

- Given a set of data from an instrument with a known limit of error, calculate the limit of error for the resulting measurement.

- Given a measurement and limit of error, write an expression with the proper significant figures that indicates the limit of error.

- Calculate the result and the limit of error when measurements with a known limit of error are added, subtracted, multiplied, or divided.

- Measure linear dimensions with a vernier caliper to the nearest 0.1 mm and with a micrometer caliper to the nearest 0.001 mm.

- Given the appropriate equipment, perform length measurements in the laboratory; and calculate the limits of error for direct and indirect measurements.

## COUNTING AND MEASUREMENT

Two distinct methods exist for determining quantities. Counting (Figure 1) allows the exact number of distinct objects to be determined. This process yields an answer in the form of an integer and involves no error if done carefully. Measurement is used to determine an unknown quantity that cannot be counted by comparison to a standard. Four types of measuring instruments are shown in Figure 2 with the quantities they are designed to measure. Even with the most accurate instruments and the greatest care, measuring always results in some error.

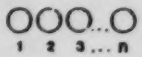



FIG. 1

An expression of a measured quantity always must contain a reference to the standard used in the measurement. An expression such as "the length of the string is 25" has no meaning. It could be 25 miles or 25 millimeters. There are three characteristics of any good standard (or unit) of measurement:

- It must be a constant. Measurement of distances in paces is not acceptable because a "pace" is different for different people.
- It must be easily reproduced and accessible. Measuring distances in terms of the wavelength of light is very accurate, but the standard unit and techniques are not available to most people.
- It must be understood and agreed to by everyone involved. Lack of understanding is the problem that must be overcome in the United States in converting to the Metric system.

FIG. 2

## ACCURACY AND PRECISION IN MEASUREMENT

"Precision" refers to the repeatability of a measurement. If a measurement of the same quantity is repeated several times with the same instrument and with perfect precision, the result will be the same each time. Accuracy refers to the agreement of the measured value with the actual or true value. These concepts are illustrated in Table I.

| TABLE 1. LENGTH OF 25-cm ROD AS MEASURED BY THREE STUDENTS | | | |
|---|---|---|---|
| Trial | Student 1 | Student 2 | Student 3 |
| 1 | 24.8 cm | 26.2 cm | 25.0 cm |
| 2 | 22.5 cm | 26.1 cm | 25.1 cm |
| 3 | 27.8 cm | 26.2 cm | 24.9 cm |
| 4 | 22.0 cm | 26.3 cm | 25.1 cm |
| 5 | 23.5 cm | 26.2 cm | 25.0 cm |
| Evaluation | Not precise or accurate | Precise but not accurate | Precise and accurate |

Three students referred to in Table 1 have made five measurements each of the length of a rod having a true length of 25 cm. The values obtained by student 1 vary greatly; therefore, this measurements are not precise, and if precision is lacking, accuracy is in doubt. Student 2 has acheived greater precision, but has obtained results that are not accurate. Precision does not assure accuracy. Student 3 has achieved both accuracy and precision in his measurements.

The accuracy and precision of measurements are determined by two classes of errors. Systematic errors usually are determinable and are due to assigned causes. They affect the accuracy of a measurement, but not the precision. Examples are a speedometer that consistently indicates a higher speed than the true value, a warped meter stick, and a constant parallax error in reading a voltmeter. Random errors usually are erratic and unpredictable and cannot always be assigned to known causes. They affect the precision of a measurement, but not necessarily the accuracy. Examples of random errors are power surges in electronic measuring equipment, changes in parallax in reading scales, and inability to repeat any measurement exactly the same way twice. Both types of errors are always present in any measurement. Reliable results require that the errors be as small as practical and that the amount of error be known and specified.

## SPECIFYING ERRORS IN MEASUREMENT

The instrument limit of error (I.L.E.) is the systematic error built into the measuring device. The magnitude of this error depends upon the manufacturing process used. The instrument limits of error for several common length-measurement devices are given in Table 2. If no limit of error is specified for a device, it generally is assumed to be one-half the smallest scale division. Electronic meters normally are assumed to have an accuracy of 2% of the full-scale reading unless otherwise specified.

| TABLE 2. ERRORS OF INSTRUMENTS FOR MEASURING LENGTH | |
|---|---|
| Measuring Instrument | Instrument Limit of Error |
| Wooden meter stick | 0.4 mm |
| Plastic scale (linear) | 0.2 mm |
| Metal scale (linear) | 0.1 mm |
| Vernier Caliper | 0.05 mm |
| Micrometer caliper | 0.005 mm |

Random errors are analyzed from calculation of the statistical limit of error (S.L.E.) from Equation 1.

$$\text{Statistical limit of error} = \frac{4 \times \text{average deviation}}{\sqrt{\text{numer of measurements}}}$$

$$\text{In Symbols:} \qquad \text{S.L.E.} = \frac{4\ a.d.}{\sqrt{n}} \qquad (1)$$

The use of this equation is illustrated by Example A.

**EXAMPLE A: CALCULATION OF STATISTICAL LIMIT OF ERROR**

Given: Nine readings of length are listed in the below table.

Find: Length and statistical limit of error.

Solution:

| | Readings (cm) | Deviations (cm) |
|---|---|---|
| | 96.44 | 0.13 |
| | 96.12 | 0.19 |
| | 96.28 | 0.03 |
| | 96.34 | 0.03 |
| | 96.45 | 0.14 |
| | 96.38 | 0.07 |
| | 96.11 | 0.20 |
| | 96.38 | 0.07 |
| | 96.30 | 0.01 |
| Total | 866.80 cm | 0.87 cm |
| Average | 96.31 cm | 0.097 cm |

Compute average deviation:

1. Total all readings.
2. Divide toal by number of readings to find average (96.31 cm).
3. Find difference between each reading and the average. List all deviations as positive quantities; for example —

   (96.44 - 96.31 = 0.13)
   (96.31 - 96.12 = 0.19)

4. Total deviations.
5. Divide total deviation by number of readings for average deviation.

$$\text{S.L.E.} = \frac{4\ \text{a.d.}}{\sqrt{n}} \qquad \text{(Equation 1)}$$

$$= \frac{(4)\ (0.097\ \text{cm})}{\sqrt{9}}$$

$$= \frac{0.388\ \text{cm}}{3}$$

$$\text{S.L.E.} = 0.13\ \text{cm}$$

Result: Length = 96.31 ± 0.13 cm

The result obtained in Example A indicates that there is a good chance (something like 600:1 odds) that the true length of the object being measured is between 96.18 and 96.44 cm (written as 96.31 ± 0.13 cm) if there is no error in the measuring device itself (which never is the case). The limit of error (L.E.) for the measurement must include both the statistical limit of error and the instrument limit of error, as shown in Equation 2.

Limit of Error = Statistical limit of error + Instrument limit of error

In Symbols: $\text{L.E.} = \text{S.L.E.} + \text{I.L.E.}$ (2)

The use of this equation is shown in Example B.

**EXAMPLE B: CALCULATION OF LIMIT OF ERROR**

Given: A length of 96.31 ± 0.13 cm is measured.
I.L.E. = 0.4 mm (wooden meter stick, Table 2).

Find: True length and limit of error.

Solution: L.E. = S.L.E. + I.L.E.
= 0.13 cm + 0.04 cm
L.E. = 0.17 cm
Length = 96.31 ± 0.17 cm

The disucssion thus far has considered only "direct" measurements — those in which some quantity is compared to a standard. "Indirect" measurements are the results of calculations involving one or more quantities determined by direct measurements. The following rules apply in determining the limit of error of indirect measurements:

- **ADDITION**

Suppose C = A + B, where A = 200.1 ± 0.2 cm (direct measurement).
B = 10.2 ± 0.1 cm (direct measurement).
Then C = 210.3 ± 0.3 cm (indirect measurement).

The rule in addition simply is to add the limits of error; i.e., (±0.2 cm) + (±0.1 cm) = ±0.3 cm.

- **SUBTRACTION**

Suppose C = A - B, where A = 200.1 ± 0.2 s, and B = 10.2 ± 0.1 s.
Then C = 189.9 ± 0.3 s.

The rule in subtraction is the same as for addition. Although the mean values are subtracted (200.1 s - 10.2 s), the limits of error (±0.2 s and ±0.1 s) are added.

- **MULTIPLICATION**

Suppose c = A x B, where A = 200 ± 2 m, or A = 200 m ± 1%,
since 2/200 = 1%.
B = 10 ± 0.4 m, or B = 10 m ± 4%,
since 0.4/10 = 4%.
Then C = (200 m x 10 m) ± 1% ± 4%
= 2000 $m^2$ ± 1% ± 4%
= 2000 $m^2$ ± 5%

Notice that the limits of error for both measurements A and B first were converted to a relative or percentage limit of error. That is, A = 200 ± 2 m, where 200 m is the mean value and ±2 m is the limit of error, was written as 200 m ± 1%. Similarly, B became 10 m ± 4%. C was obtained by multiplying the mean values directly to obtain 2000 $m^2$ and adding the relative or percentage errors to obtain ±5%. Thus, C was given as 2000 $m^2$ ± 5%. Now, 5% is a relative error. If C is to be written in terms of an absolute limit of error, it becomes C = 2000 $m^2$ ± 5% of 2000 $m^2$, or C = 2000 ± 100 $m^2$. Thus, the rule for propagation of errors in multiplication is to add the relative or percentage errors associated with each direct measurement.

- DIVISION

Suppose C = A/B, where A = 200 ± 2 m = 200 m ± 1%.

B = 10 ± 0.4 s = 10 s ± 4%.

Then, C = 200 m/10 s ± 5% = 20 m/s ± 5% = 20 ± 1 m/s.

The rule is the same as that for multiplication. The errors first are converted to relative or percentage errors and then added to obtain the overall percentage error associated with the quotient.

- POWERS AND ROOTS

Since a number raised to a given power, such as $C = A^3$, is an example of multiplication, the relative error for C would be three times the relative error for A. Likewise, for a number such as $C = A^{1/3}$, the relative error for C would be 1/3 times the relative error for A.

In reporting the results of a measurement the numerical value must be expressed with the proper number of significant figures. In Example A a meter stick was used to measure a length. The smallest division on a meter stick is 0.1 cm, and all readings were estimated to the nearest tenth of that division, or 0.01 cm. Calculations with a pocket calculator give a result of 96.311111 ± 0.1288889 cm. Writing all these digits indicates greater accuracy than actually was achieved. The final answer always is rounded off to leave the same number of digits to the right of the decimal as the original readings, in this case 96.31 ± 0.13 cm. All calculations preferably should be completed before the answer is rounded off, to avoid the introduction of additional errors.

When indirect measurements are calculated from two or more direct measurements with different significant figures, the answer should be expressed with the same number of significant figures as the direct measurement with the lowest number of significant figures, as indicated in the velocity calculation of Example C.

**EXAMPLE C: VELOCITY CALCULATION**

**Given:** $\text{Velocity} = \frac{\text{distance}}{\text{time}} = \frac{d}{t}$

distance: $d = 85.15 \pm 0.12$ cm
time: $t = 22 \pm 1$ s

**Find:** Velocity and limit of error.

**Solution:** $d = 85.15 \pm 0.12 \text{ cm} = 85.15 \text{ cm} \pm 1.41\%$
$t = 22 \pm 1 \text{ s} = 22 \text{ s} \pm 4.5\%$
$v = \frac{d}{t}$
$= \frac{85.15 \text{ cm}}{22 \text{ s}} \pm (1.41\% + 4.5\%)$
$v = 3.87045 \text{ cm/s} \pm (5.91\%)$
$v = 3.9 \text{ cm/s} \pm 5.9\%$ (Round off to 2 significant figures.)
$v = 3.9 \pm 0.2 \text{ cm/s}$

## THE READING OF SCALES

When reading linear scales, the smallest unit to which a measurement can be estimated is one-tenth the smallest scale division. For scales with small divisions, the practical limit may be only one half of a division.

Figure 3 shows a vernier caliper that affords greater accuracy. This device incorporates two scales, as indicated in Figure 4. The main scale has divisions at a spacing of 0.1 cm. The vernier scale is 10% smaller. When a length is measured, the vernier scale slides along beside the fixed scale. The first two significant figures of the length are determined by the position of the zero mark on the vernier scale; the third is the vernier scale mark that lines up with any mark on the main scale. The length in Figure 4 is 2.00 cm. Figure 5 is another example. The zero mark of the vernier scale is just past the 2.3 mark on the main scale, and the 0.06 mark of

FIG. 3

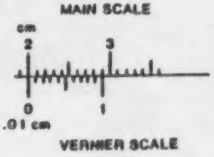



FIG. 4

vernier scale lines up with a mark on the main scale. The measured length is 2.36 cm.

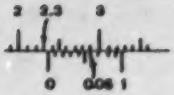



FIG. 5

The micrometer caliper illustrated in Figure 6 is still more accurate. Figure 7 displays the scales of a micrometer calibrated in ten thousandths of an inch. The smallest main-scale divisions are 0.025 in, the thimble-scale divisions are 0.001 in, and the vernier scale divisions are 0.0001 in. The reading in Figure 7 is 0.2500 in.

FIG. 6

Figure 8 is another example of a micrometer reading. The components of the indicated length are as follows:

| | |
|---|---|
| main scale – | 0.1750 in |
| thimble scale – | 0.0110 in |
| vernier scale – | 0.0004 in |
| Total | 0.1864 in |

Some micrometers lack the vernier scale. These are read to the nearest thousandths of an inch (or .001 cm for metric micrometers) on the thimble scale. The next digit may be estimated if desired.

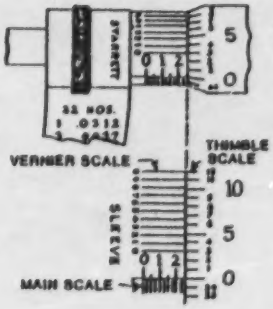



FIG. 7

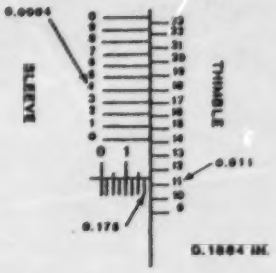



FIG. 8

## SUMMARY

In the technological world of today, precision measurement is a strict requirement. "Precision" refers to the repeatability of a measurement; "accuracy," to the agreement of the measured value with the actual, or true, value.

Systematic errors in measurement result from the imperfections of equipment; whereas, random errors result from imperfect observation or technique. The instrument limit of error (I.L.E.) is the systematic error built into the measuring device. Significant figures must be used to express a numerical value with accuracy.

The use of the vernier caliper, of the micrometer, and of the variety of other scales and devices is important and must be understood by the technician.

## LENGTH MEASUREMENTS AND LIMITS OF ERROR

### MATERIALS

Wooden meter stick.
Vernier caliper (metric).
Micrometer caliper (metric).
Metal cylinder, approximately 1.20 cm (½ inch) in diameter and 1.90 cm (3/4 inch) in length.

### PROCEDURE

1. Use meter stick to make 9 diameter measurements and 9 length measurements of the cylinder. Record values in Data Table 1 to the nearest 0.01 cm.

2. Record the instrument limit of error from Table 2 of Discussion.

3. Complete calculations in Data Table 2 to determine volume of cylinder and limit of error of volume.

4. Use the vernier caliper and then the micrometer caliper to make 9 measurements of the diameter of the cylinder. Read each instrument for the greatest possible accuracy. Record values in Data Table 3. (Refer to Figure 8).

5. In Data Table 4, record the instrument limits of error from Table 2 of Discussion.

6. Calculate the diameter and limit of error for each set of measurements; record in Data Table 3.

7. Compare values obtained for diameter and limit of error with the three measuring devices. If different values are obtained with the three devices, explain why.

| DATA TABLE 1 | | | | |
|---|---|---|---|---|
| METER STICK I.L.E. = _____ | | | | |
| Trial | length (cm) | deviation (cm) | diameter (cm) | deviation (cm) |
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |
| 9 | | | | |
| Totals | | | | |
| Average | | | | |

**DATA TABLE 2**

**LENGTH**

Statistical Limit of Error: $S.L.E. = \frac{4\ a.d.}{n}$

$= (4)\ (\_\_\_\_\ cm) \div 9$

$= \_\_\_\_\_\ cm \div 3$

$S.L.E. = \_\_\_\_\_\ cm$

Limit of Error: $L.E. = S.L.E. + I.L.E.$

$= \_\_\_\_\_\ cm + \_\_\_\_\_\ cm$

$= \_\_\_\_\_\ cm$

Measured Length: $l = \_\_\_\_\_ \pm \_\_\_\_\_ cm$

**DIAMETER**

Statistical Limit of Error: $S.L.E. = \_\_\_\_\_ cm$

Limit of Error: $L.E. = \_\_\_\_\_ cm$

Measured Diameter: $d = \_\_\_\_\_ \pm \_\_\_\_\_ cm$

**VOLUME**

Mean Volume: $V = \frac{\pi d^2 l}{4}$

$= (\frac{3.14}{4})(\_\_\_\_\_)^2(\_\_\_\_\_)$

$V = \_\_\_\_\_ \text{ cm}^3$

Limit of Error of volume

% error in diameter = $\frac{\text{L.E. of diameter}}{\text{mean diameter}}$ (100) = _____%

% error in length = $\frac{\text{L.E. of length}}{\text{mean length}}$ (100) = _____%

% error in volume = (2 x % error in diameter) + % error in length

= _____%

Limit of error in volume = mean volume x % error in volume ÷ 100

L.E. = _____ $\text{cm}^3$ x _____ ÷ 100

L.E. = _____ $\text{cm}^3$

Volume = _____ ± _____ $\text{cm}^3$

**DATA TABLE 3**

| | Vernier Caliper I.L.E. = _____ | | Micrometer Caliper I.L.E. = _____ | |
|---|---|---|---|---|
| Trial | Diameter (cm) | Deviation (cm) | Diameter (cm) | Deviation (cm) |
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |
| 9 | | | | |
| Totals | | | | |
| Average | | | | |

| DATA TABLE 4 |
| --- |
| Vernier Caliper<br>S.L.E. = _____<br>L.E. = _____<br>Length: l = _____<br><br>Micrometer Caliper<br>S.L.E. = _____<br>L.E. = _____<br>Length: l = _____ |

## SELF CHECK

1. Name two quantities commonly counted and two that are measured.

2. State the difference between precision and accuracy.

3. Name one systematic error and one random error possible in the laboratory of this module.

4. The following measurements were made of the diameter of a ball bearing with a vernier caliper. Determine the average diameter and limit of error.

| Trial | Diameter (cm) | |
| --- | --- | --- |
| 1 | 1.27 | I.L.E. from |
| 2 | 1.26 | Table 2 _____ |
| 3 | 1.27 | |
| 4 | 1.28 | |
| 5 | 1.28 | |
| 6 | 1.29 | |
| 7 | 1.27 | |
| 8 | 1.28 | |
| 9 | 1.28 | |

5. The volume of a sphere is given by $V = (4/3)\pi r^3$ or $4\pi r^3/3$. Calculate the volume of the ball bearing above and the limit of error of the volume.

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